PHYS 391
Day 18

- FFT Quiz
- Windowing

Overview

- HW5 and Lab 5 both explore aspects of the FFT
- Discrete frequency coefficients
- Nyquist limit, and aliasing
- New today: windowing

Conceptual Questions

What is the primary reason for the following features of the discrete Fourier transform?

- Finite frequency components: $\Delta\nu$
- Nyquist limit/aliasing

Conceptual Questions

- Finite frequency components: $\Delta \nu$

These were also present in the original Fourier series, where f(x) was continuous.
This comes about from the finite duration of the sampled waveform.

- Nyquist limit/aliasing

This is directly the result of the discrete nature of the sampled waveform

Sampling Parameters

If we sample a waveform at 1 kHz for 0.5 seconds,

- How many total data points will we have?
- What is the Nyquist Limit?
- What frequency spacing will we have in our Fourier coefficients?

Sampling Parameters

- How many total data points will we have?

1000 samples / second x 0.5 seconds = 500 samples

- What is the Nyquist Limit?

$\nu_{max} = \nu_s / 2 = 500$ Hz

- What frequency spacing will we have in our Fourier coefficients?

$\Delta\nu = 1/T = 2/\text{seconds} = 2$ Hz

Aliasing

If we sample a waveform at 1 kHz for 0.5 seconds,

- What frequency will a 300 Hz signal appear to have?
- What frequency will a 600 Hz signal appear to have?
- What frequency will a 900 Hz signal appear to have?
- What frequency will a 1200 Hz signal appear to have?

Aliasing

If we sample a waveform at 1 kHz for 0.5 seconds,

$\nu_{max} = \nu_s / 2 = 500$ Hz

Aliasing

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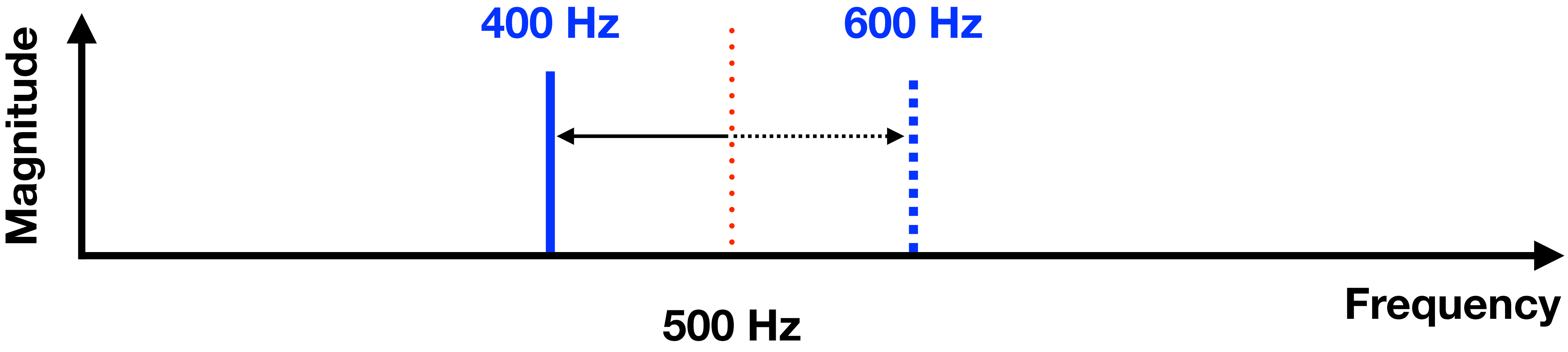
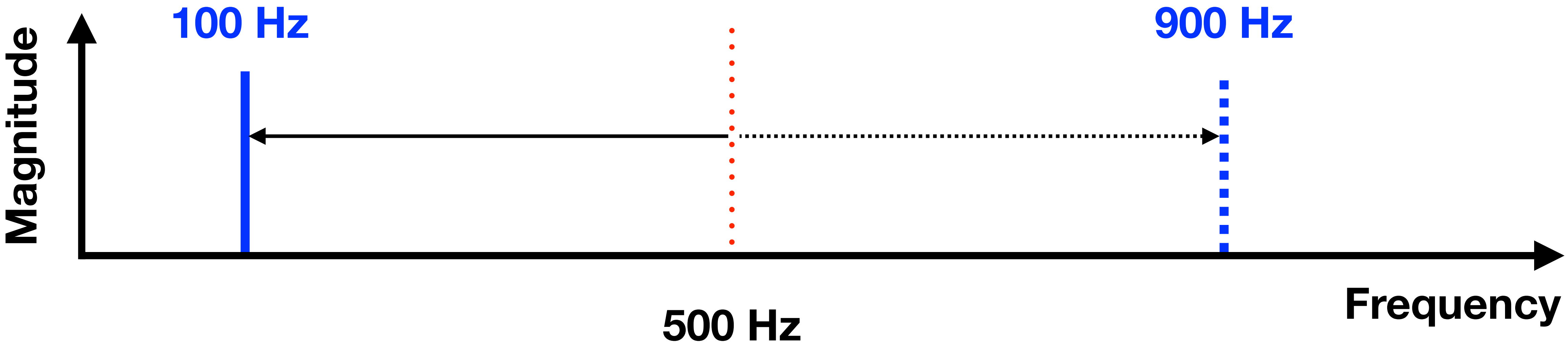


Aliasing

If we sample a waveform at 1 kHz for 0.5 seconds,

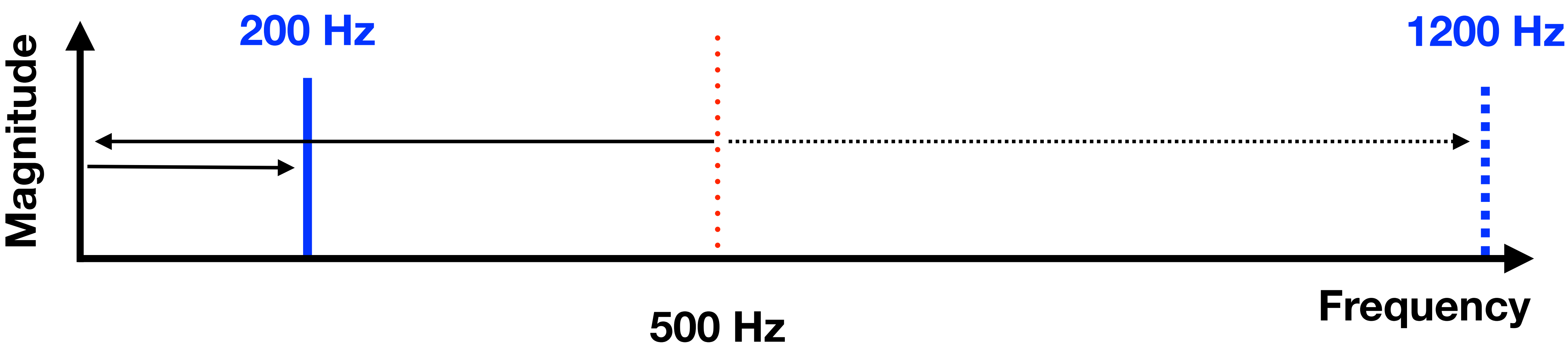
$v_{max} = v_s / 2 = 500$ Hz



Aliasing

If we sample a waveform at 1 kHz for 0.5 seconds,

$\nu_{max} = \nu_s / 2 = 500\ Hz$



No loss in amplitude!

Square Waves

We take an FFT of a square wave. The first peak in the spectrum is at 120 Hz with magnitude of 1 V.

- Where is the second peak (next highest frequency)?
- What is the amplitude of this second peak?
- Where is the 3rd peak?

You don't need to worry about aliasing here…

Square Waves

We take an FFT of a square wave. The first peak in the spectrum is at 120 Hz with magnitude of 1 V.

Odd coefficients with amplitude ~ 1/n

- Where is the second peak (next highest frequency)?

n = 1 at 120 Hz ⇒ n=3 at 360 Hz

- What is the amplitude of this second peak?

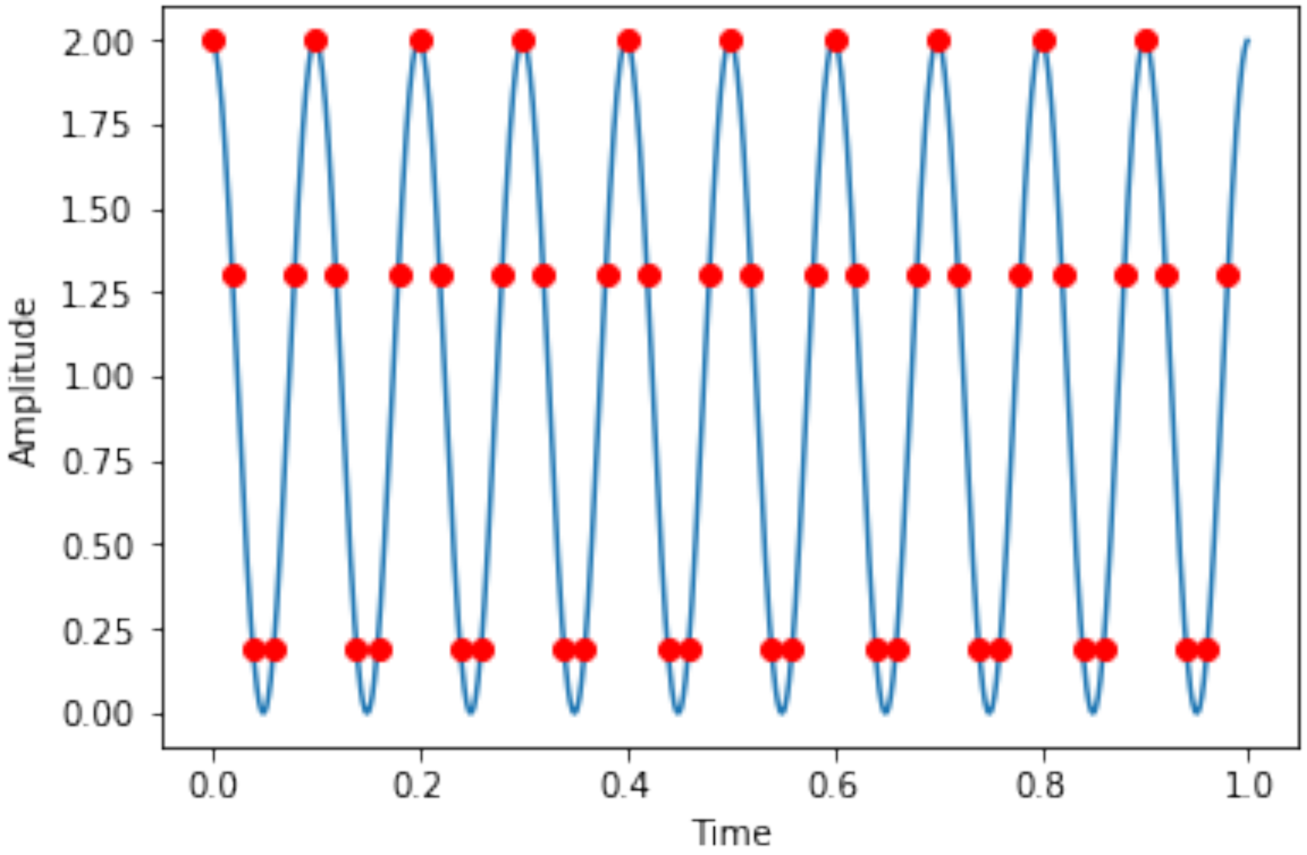
1/3 of the first peak

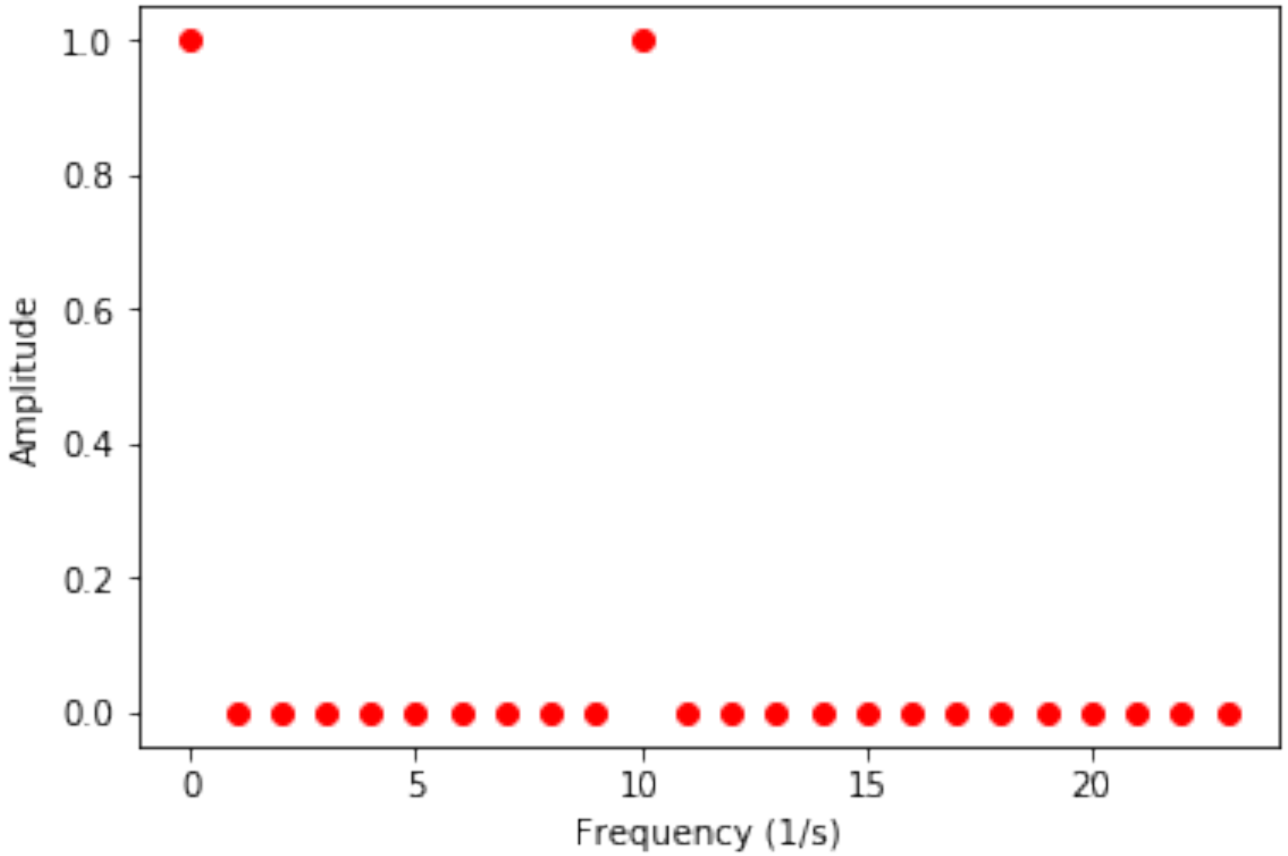
- Where is the 3rd peak?

n=5 at 600 Hz

Fourier Transform

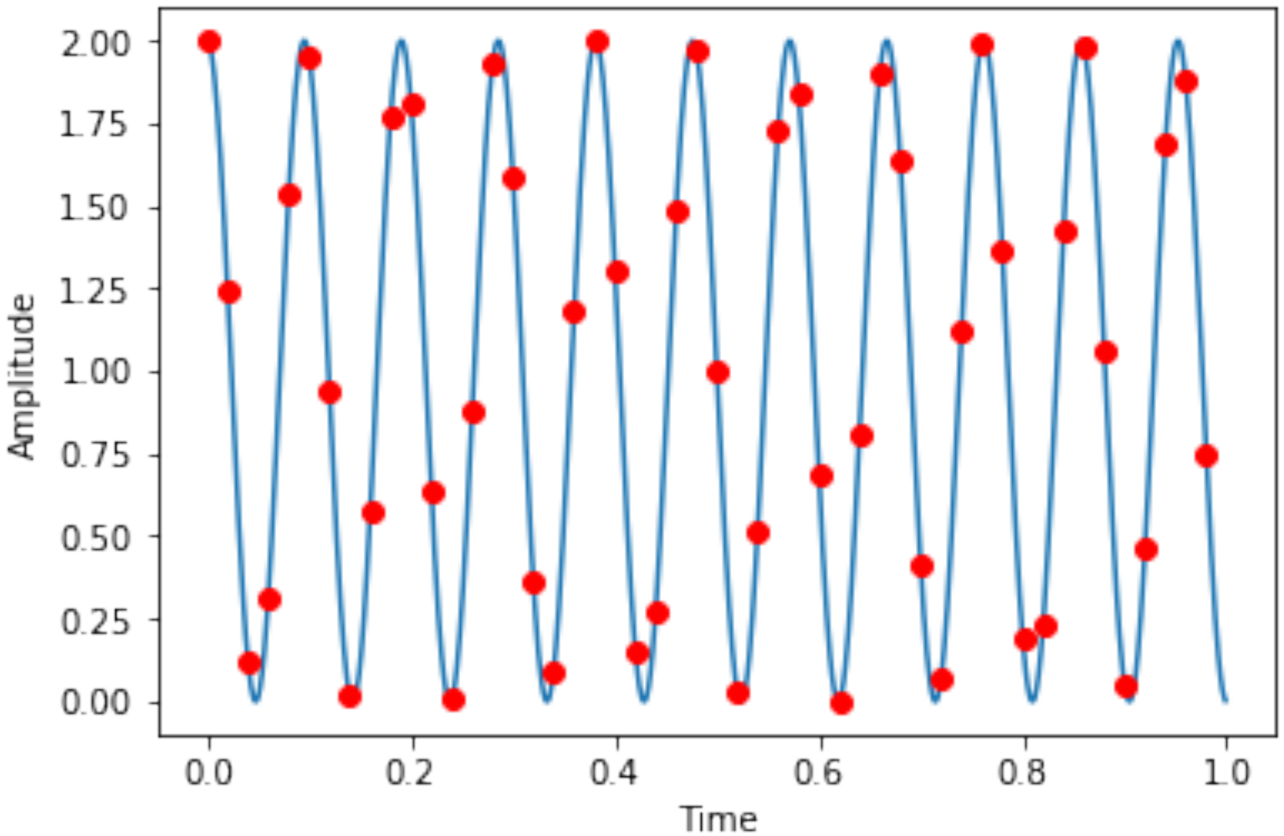
v_s = 50 Hz, v_0 = 10 Hz

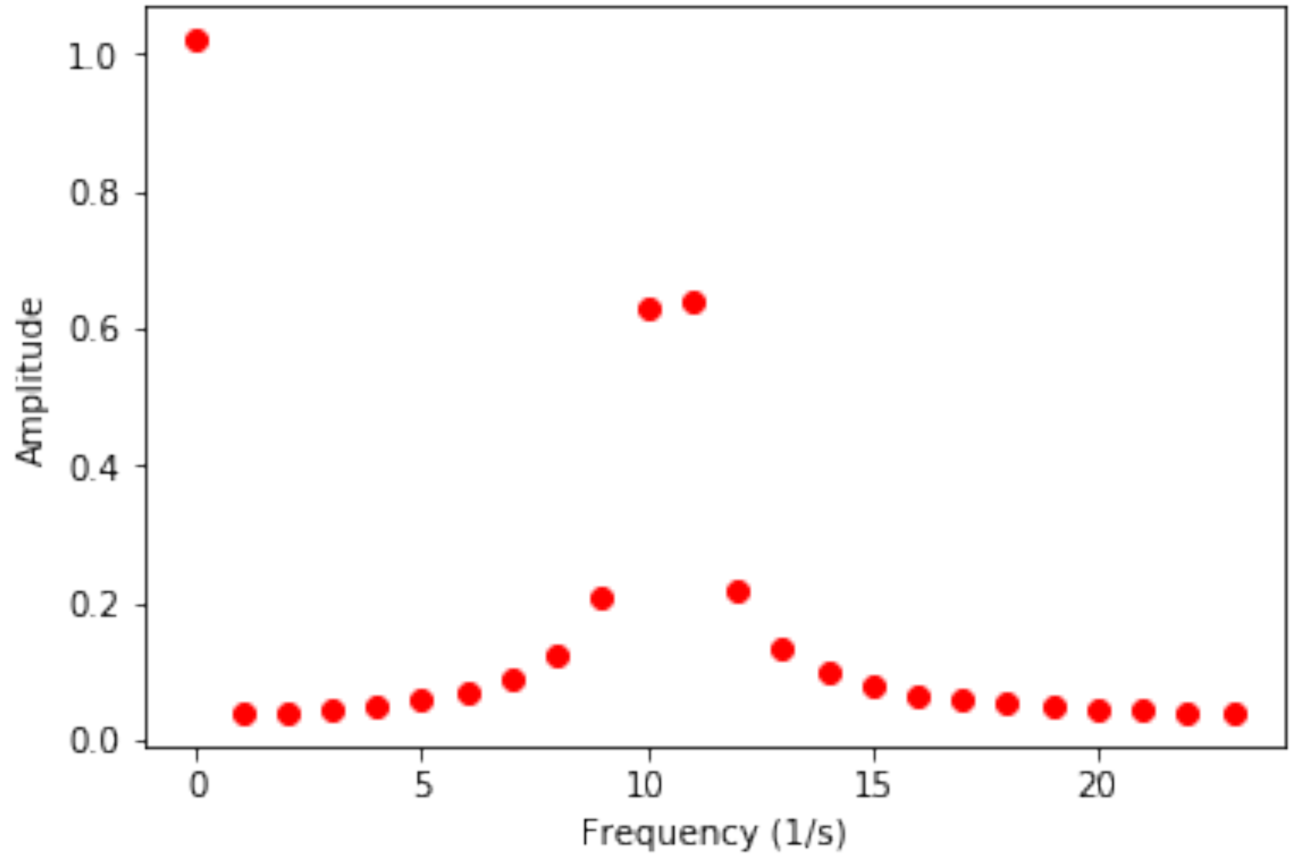




Fourier Transform

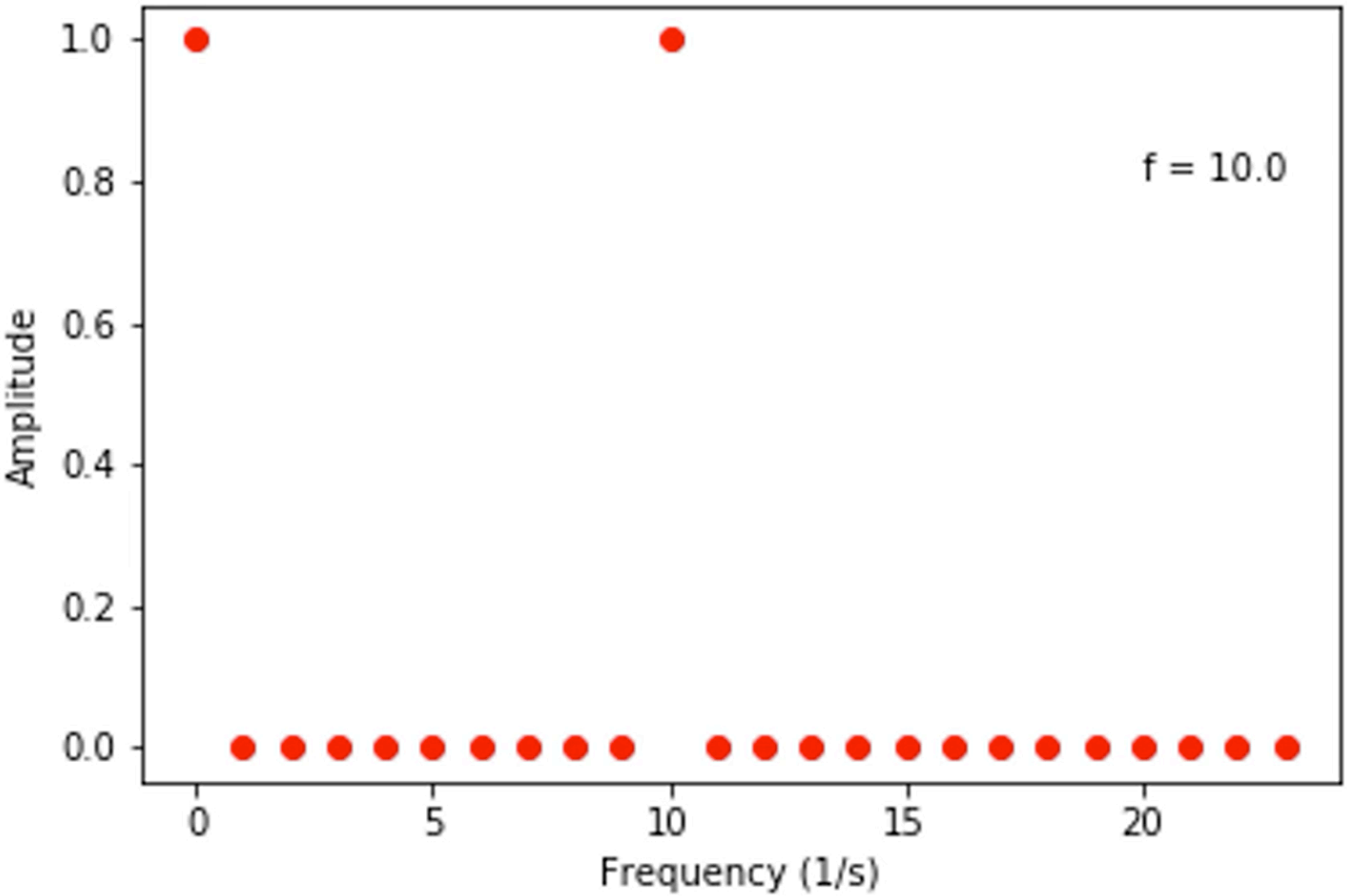
$\nu_s = 50$ Hz, $\nu_0 = 10.5$ Hz



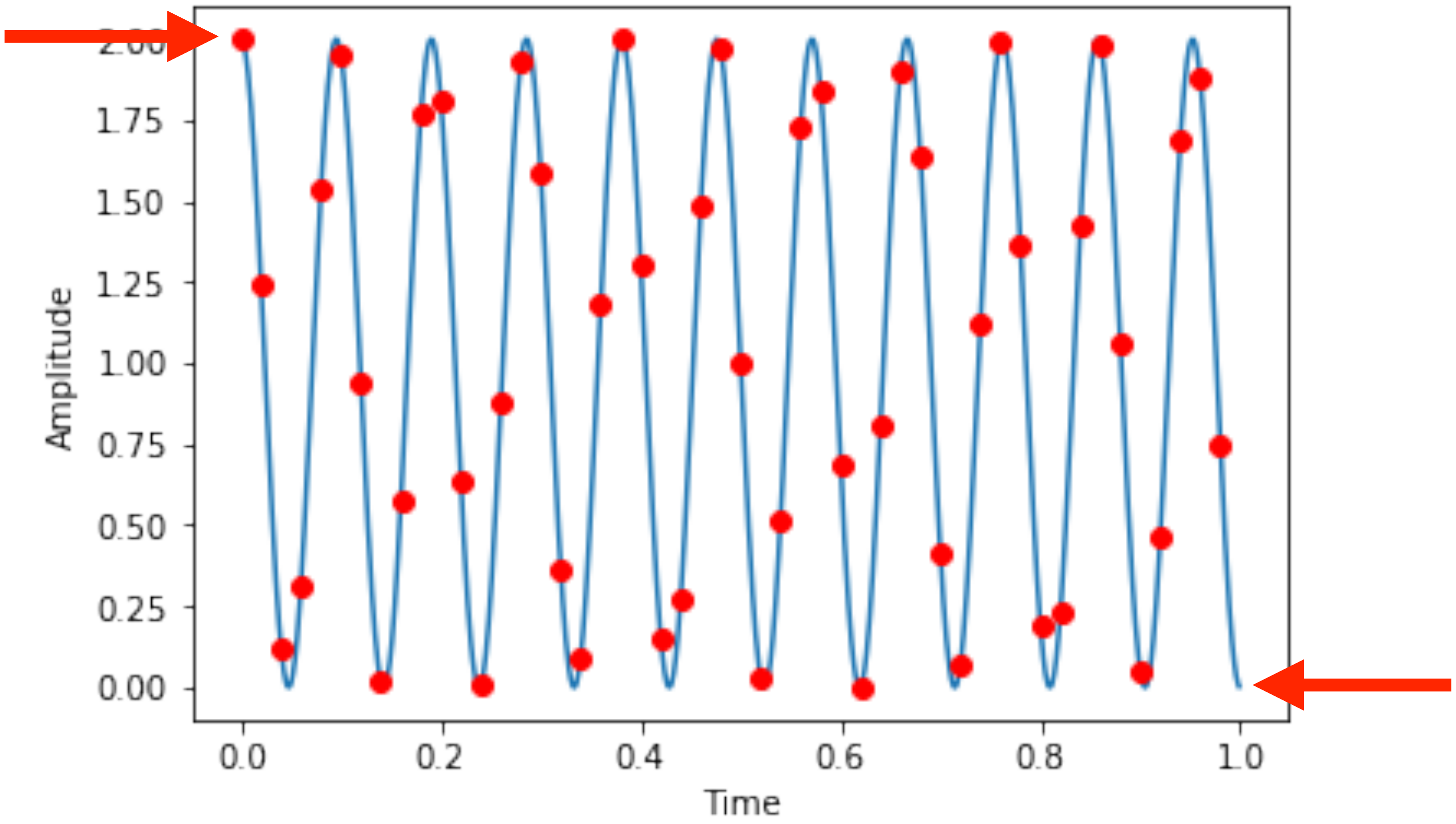


"Frequency Leakage"

Home Movies

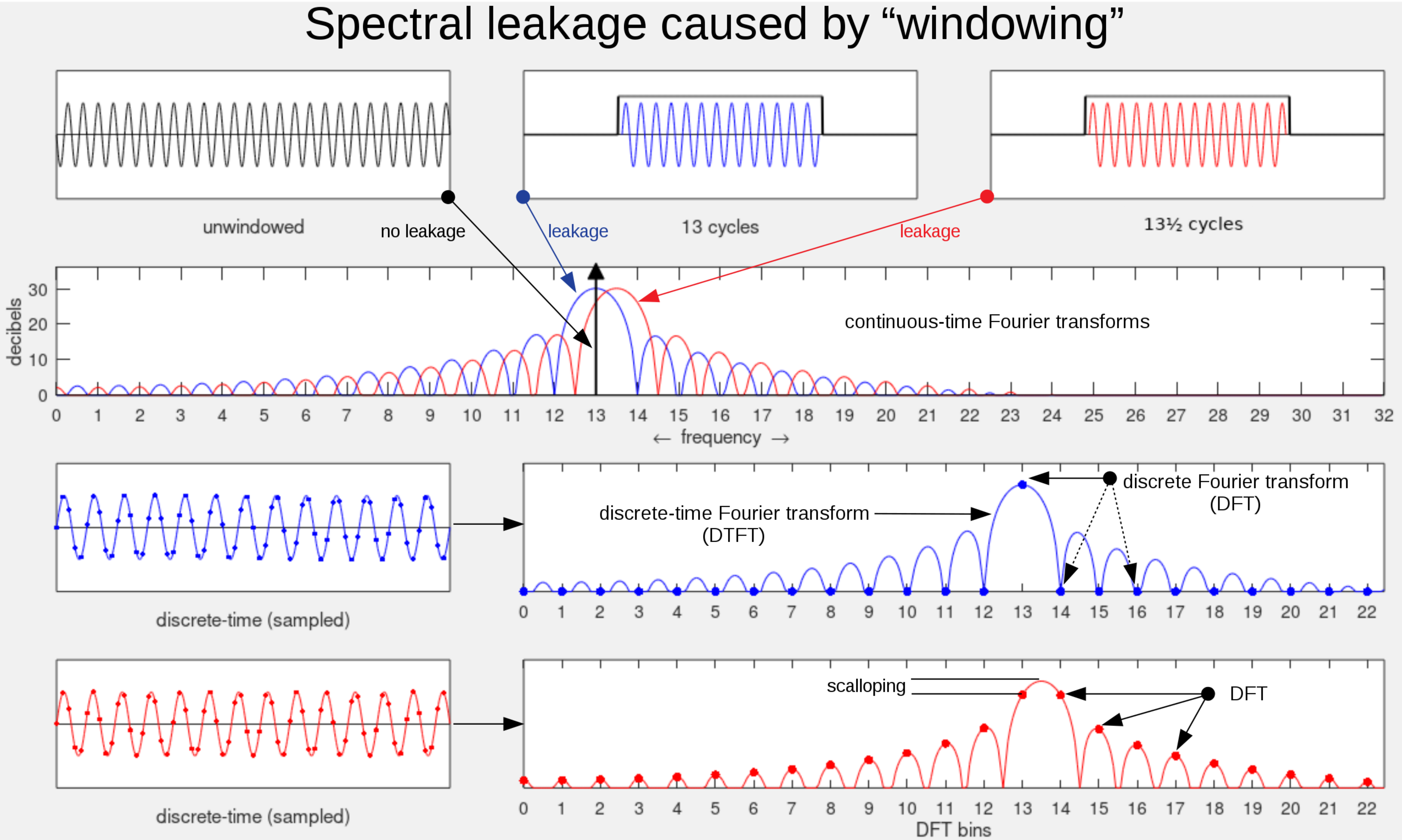


Windowing - Conceptual

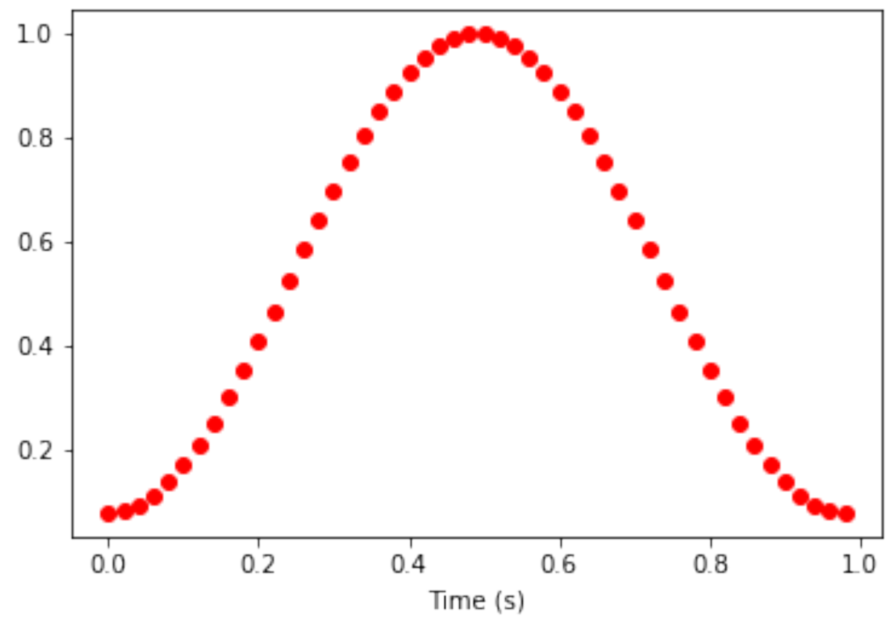


Sharp transitions lead to high frequencies

Windowing - Wikipedia

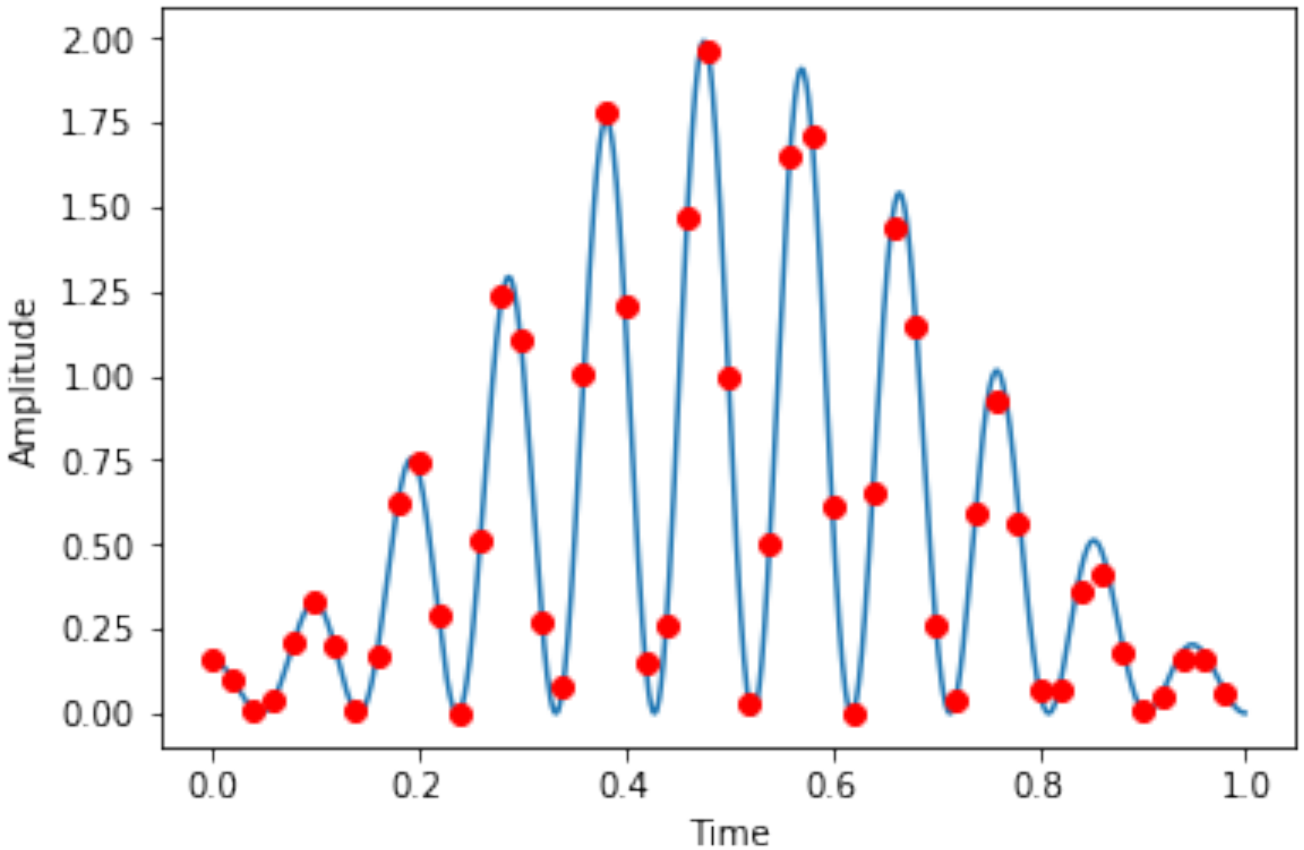


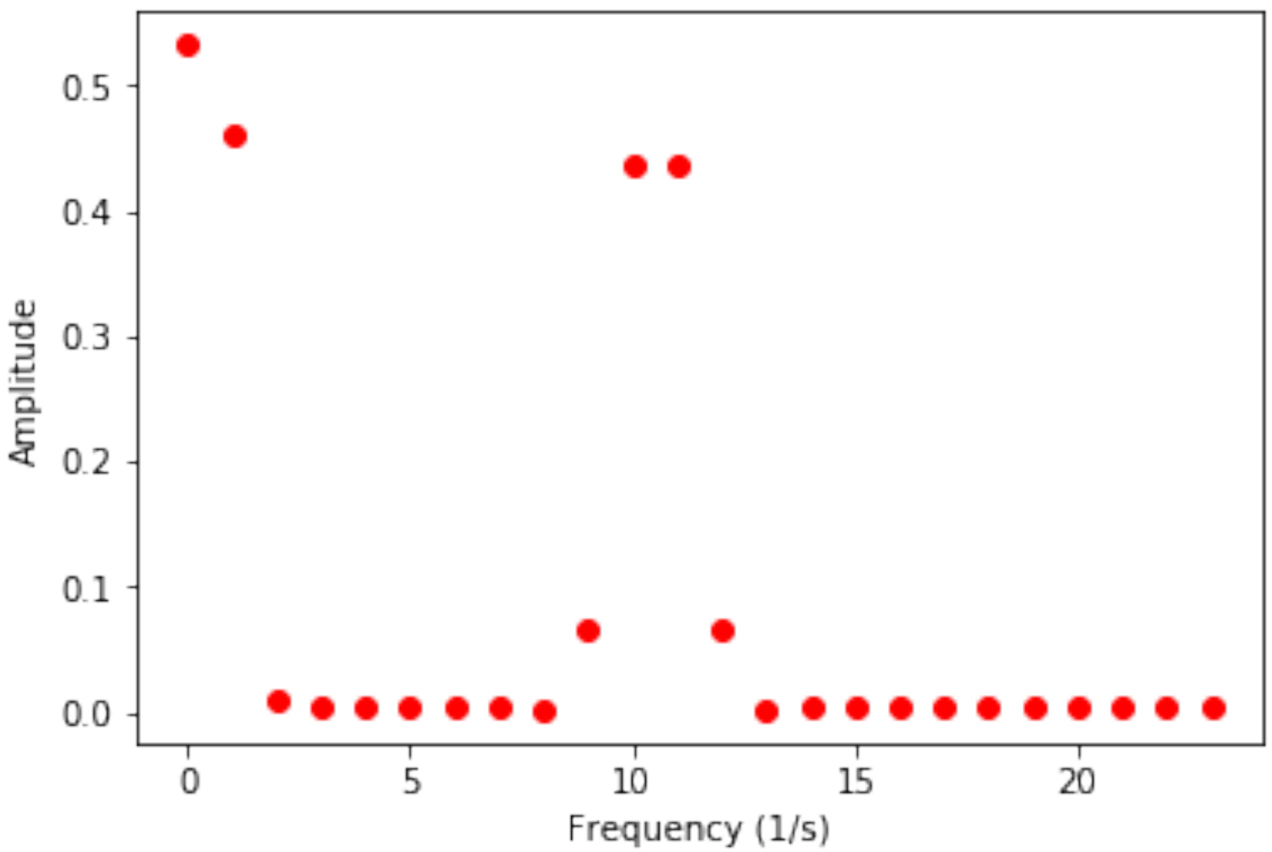
Hamming Window



Multiply time-series data by ‘window’

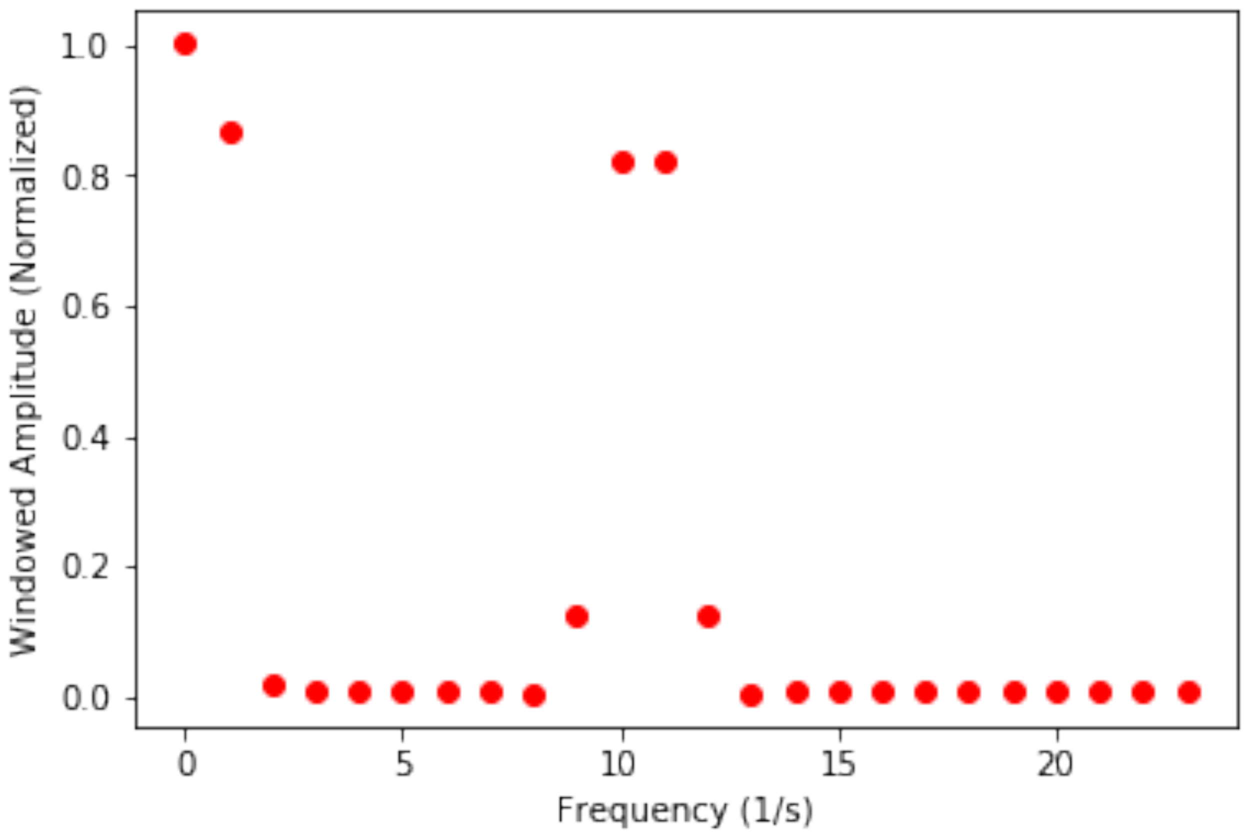
```
import scipy.signal.windows as win
n = len(y1)
window = win.hann(n)
ywindowed = y1 * window
ft1 = np.fft.fft(ywindowed)/n
```



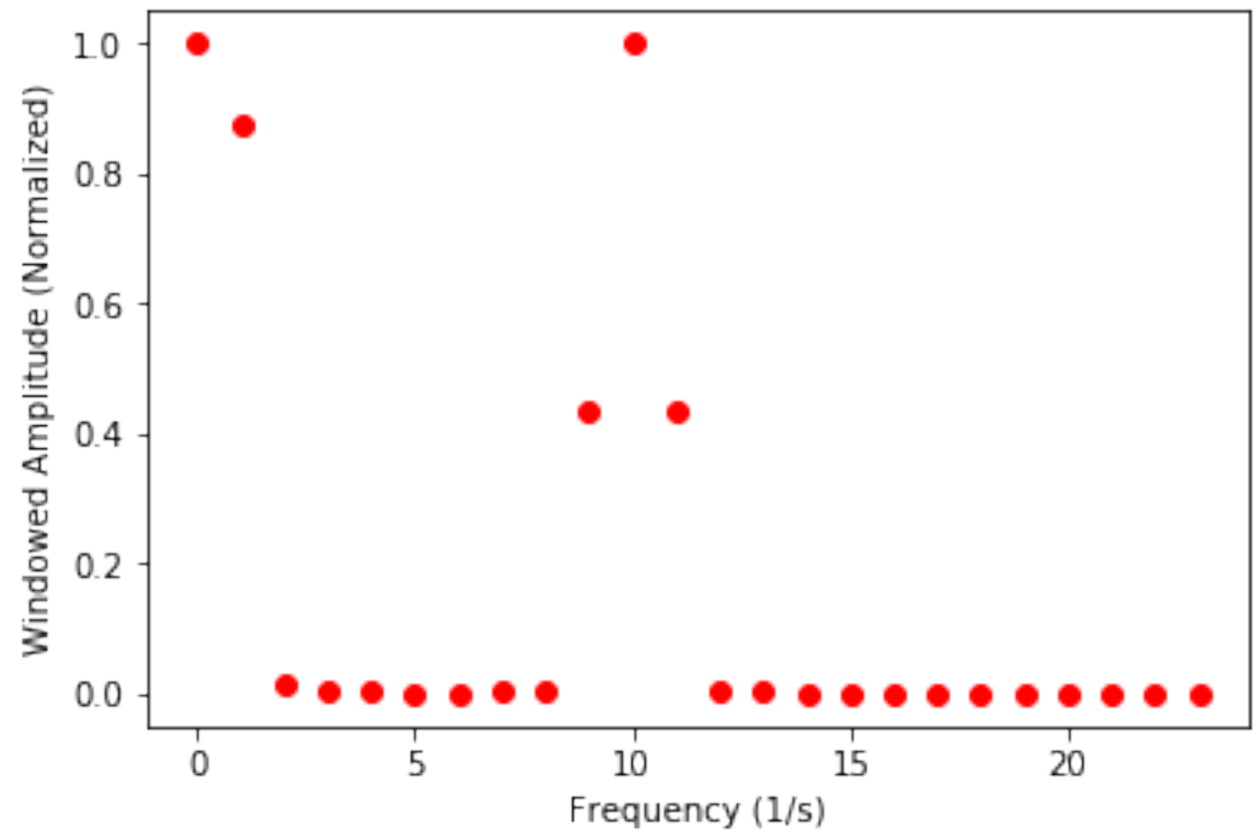


Normalized

ν_s = 50 Hz, ν_0 = 10.5 Hz



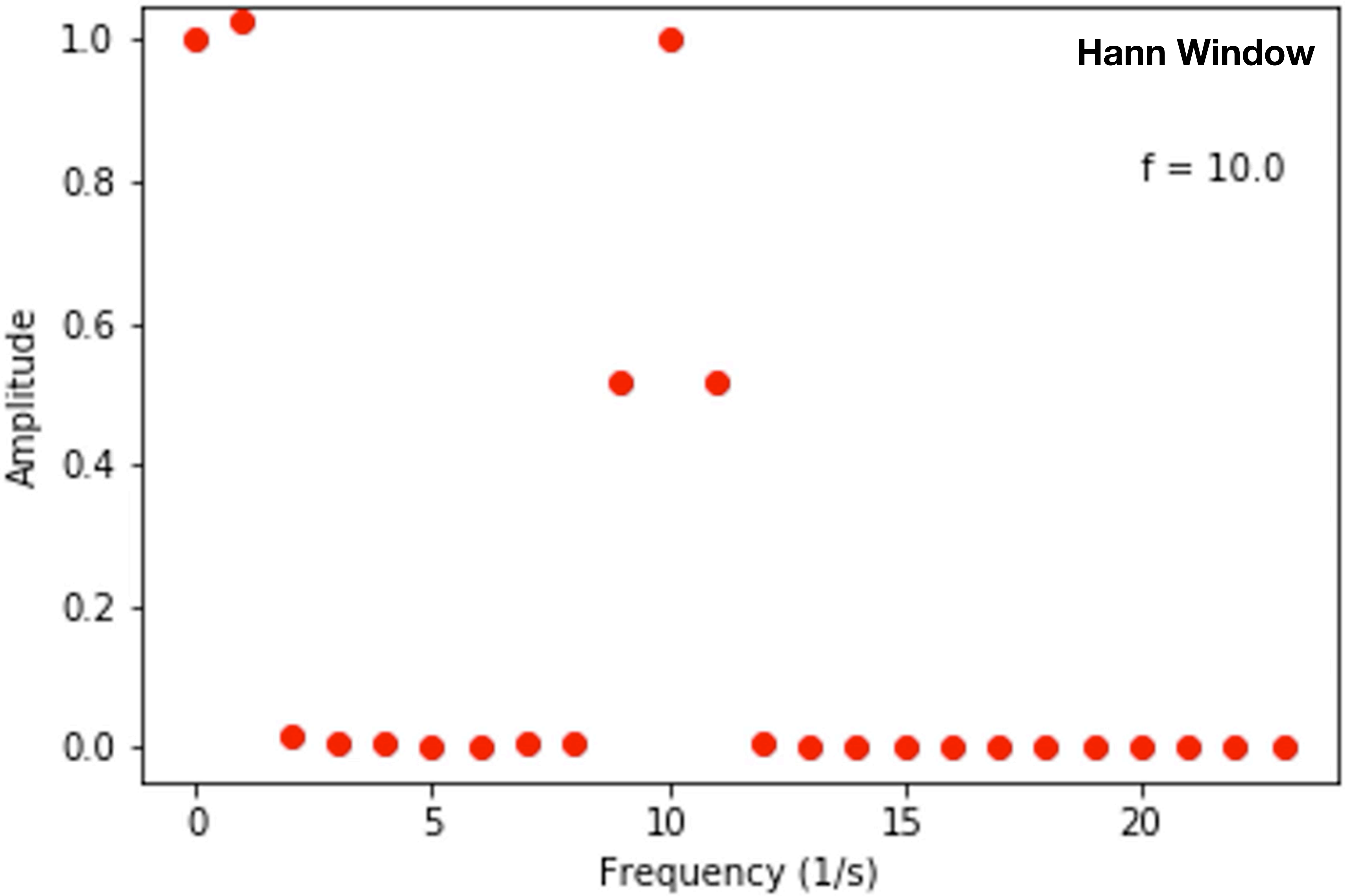
ν_s = 50 Hz, ν_0 = 10 Hz



Must correct for overall attenuation of window

```
norm = sum(window) / n
ywindowed = y1 * window / norm
```

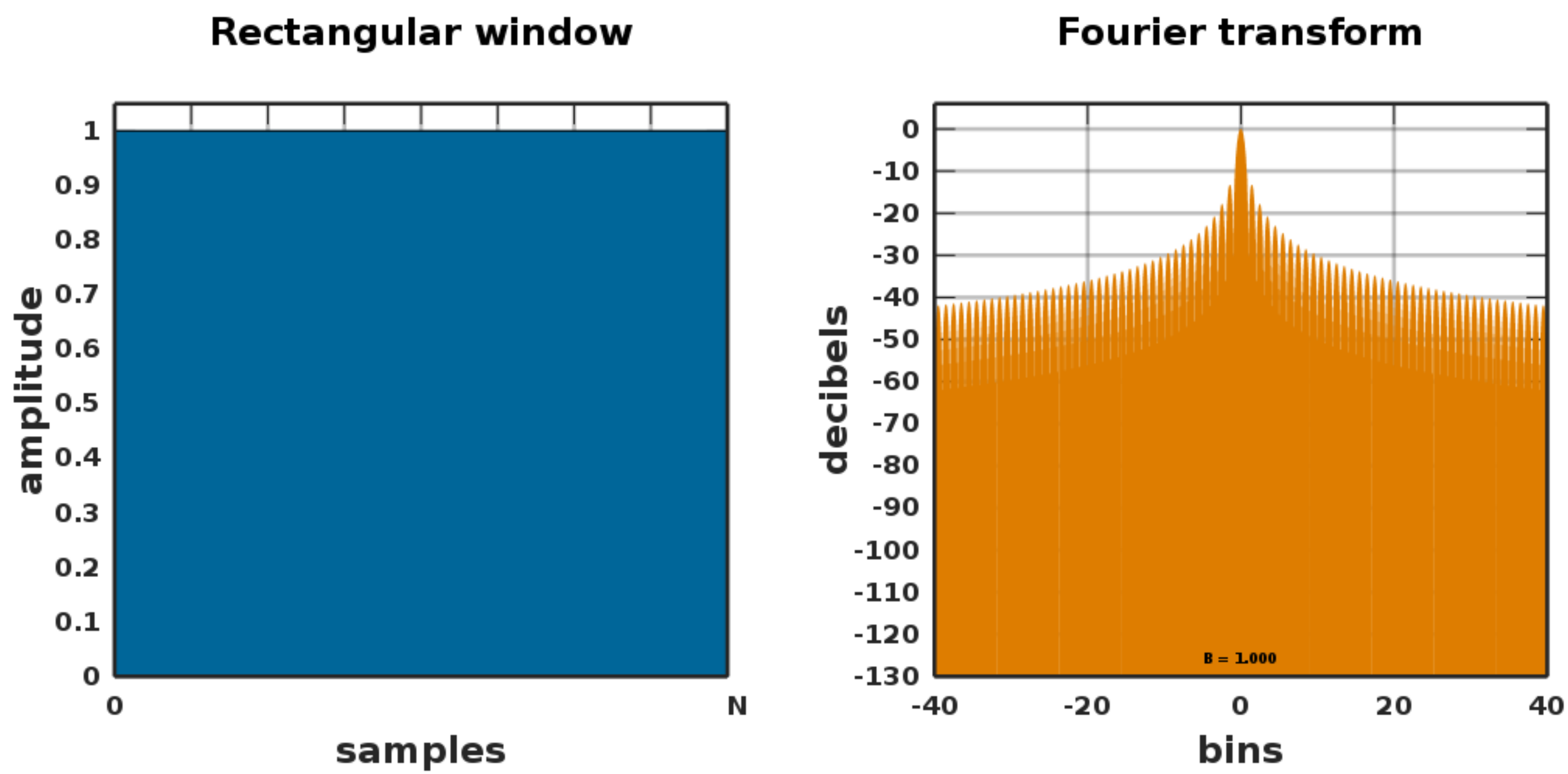
More Movies



Hann Window
f = 10.0
1.0
0.8
0.6
0.4
0.2
0.0
Amplitude
0
5
10
15
20
Frequency (1/s)

Window comparisons

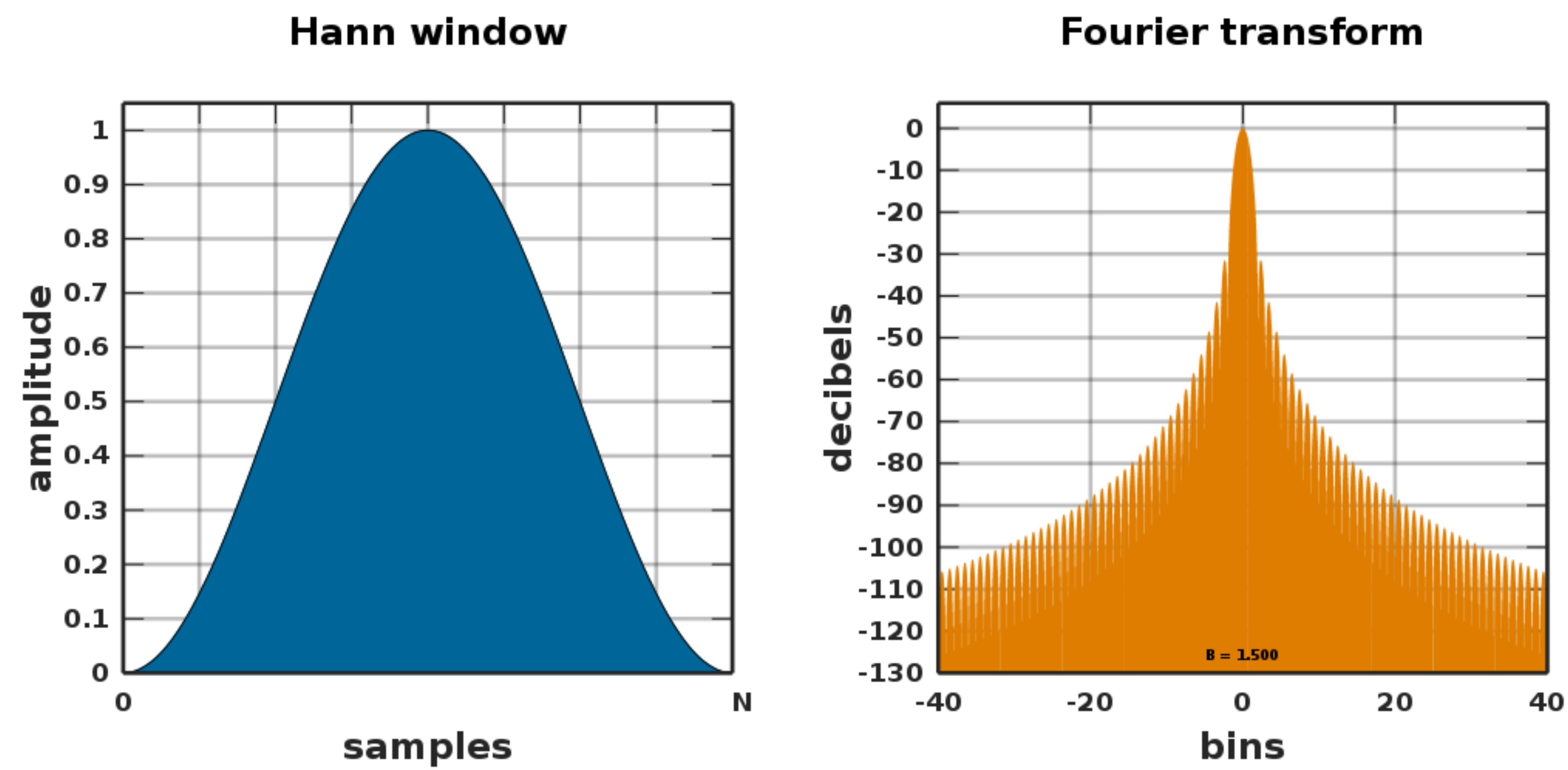
Boxcar or Rectangular Window



+10 dB = x10 in power or x20 in amplitude

Window comparisons

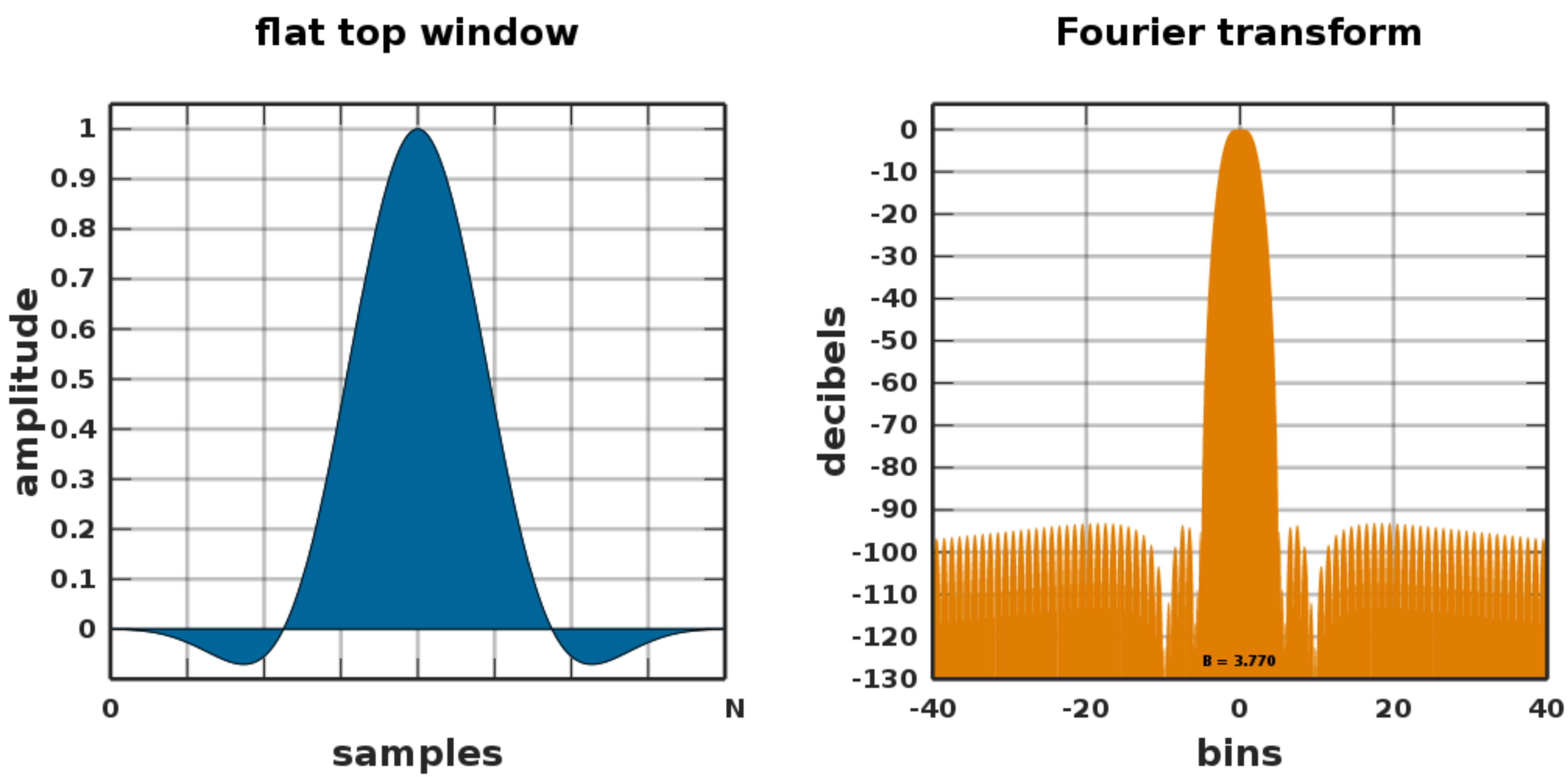
Hann Window



+10 dB = x10 in power or x20 in amplitude

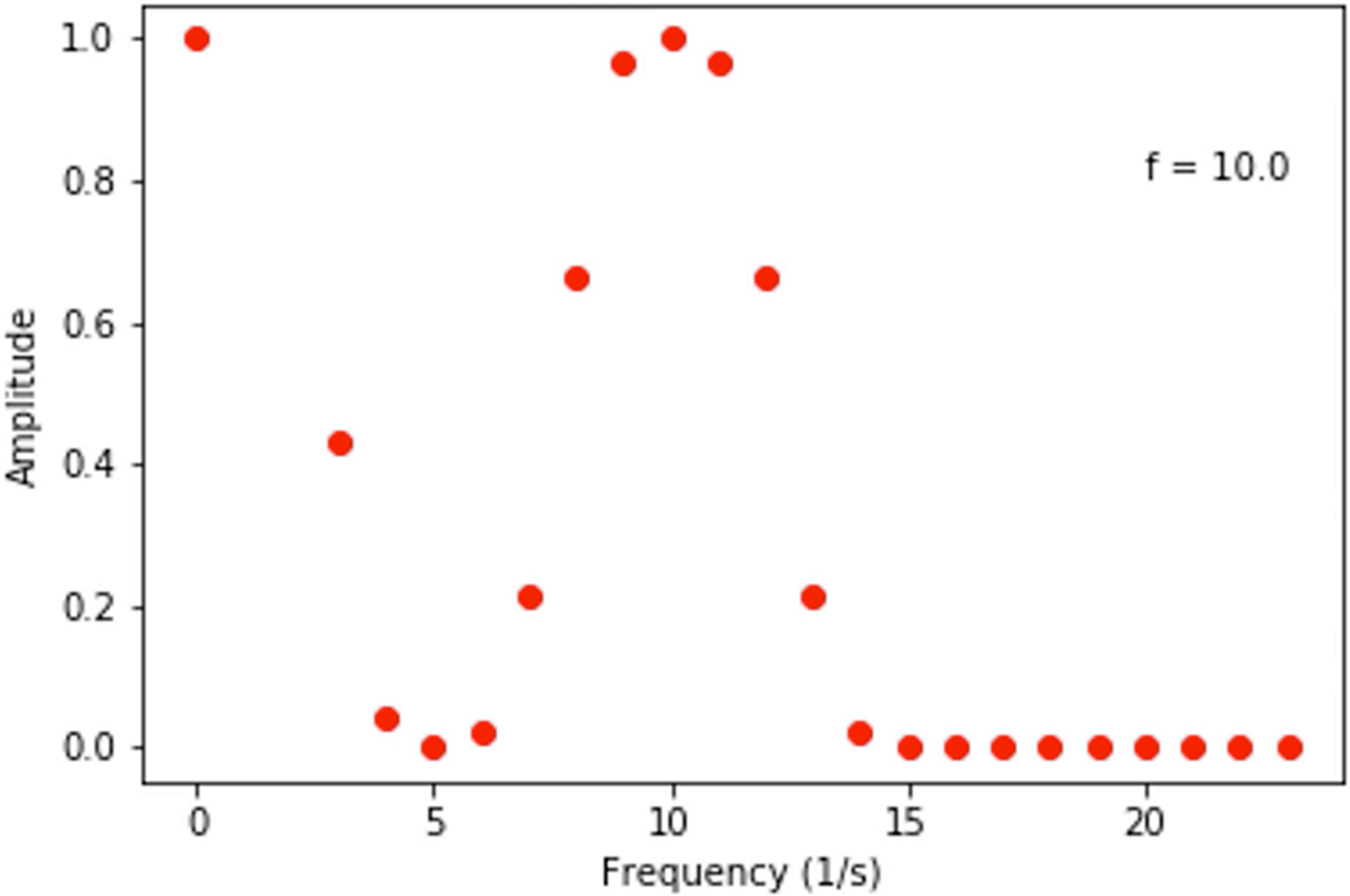
Window comparisons

Flat Top Window



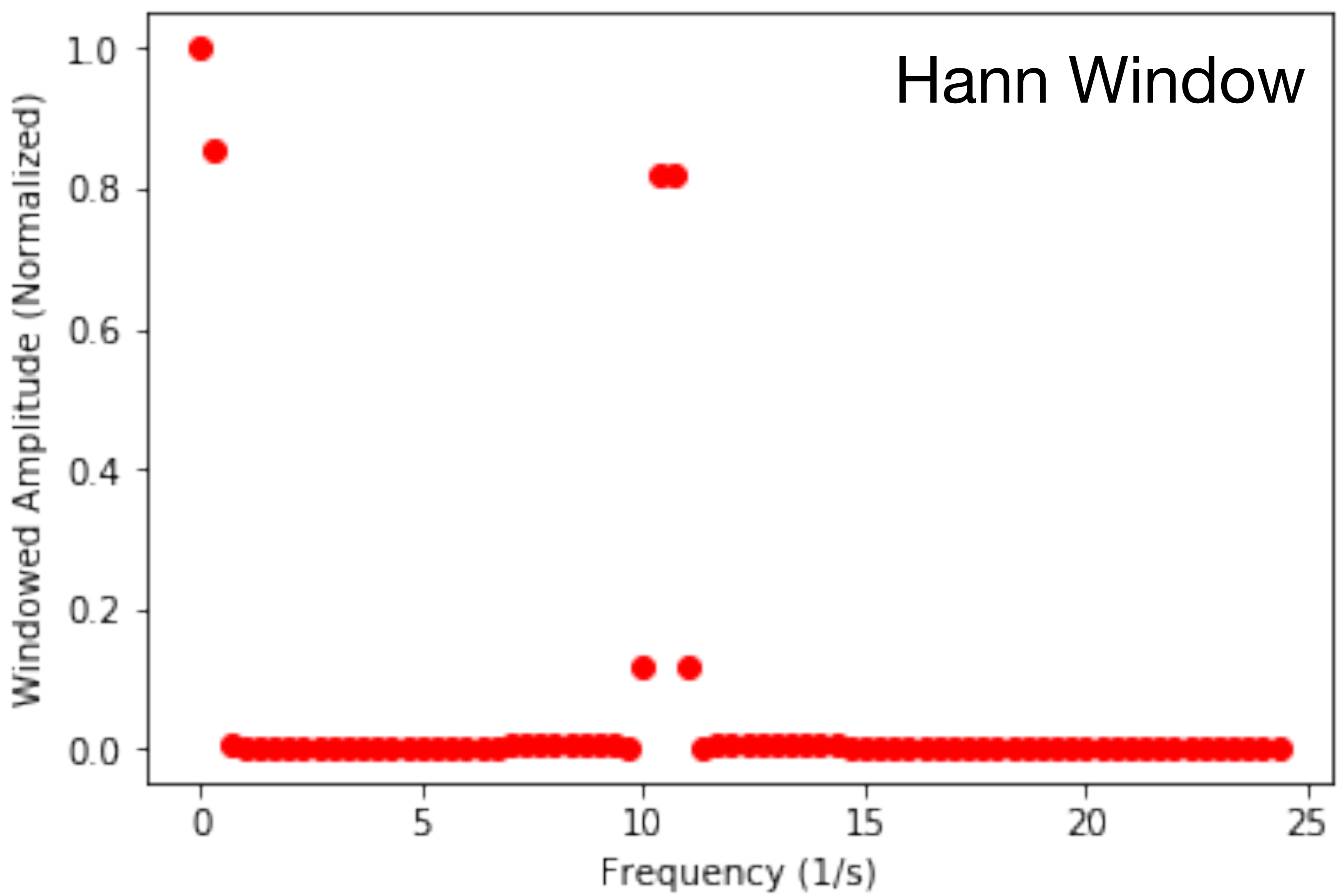
+10 dB = x10 in power or x20 in amplitude

Flattop

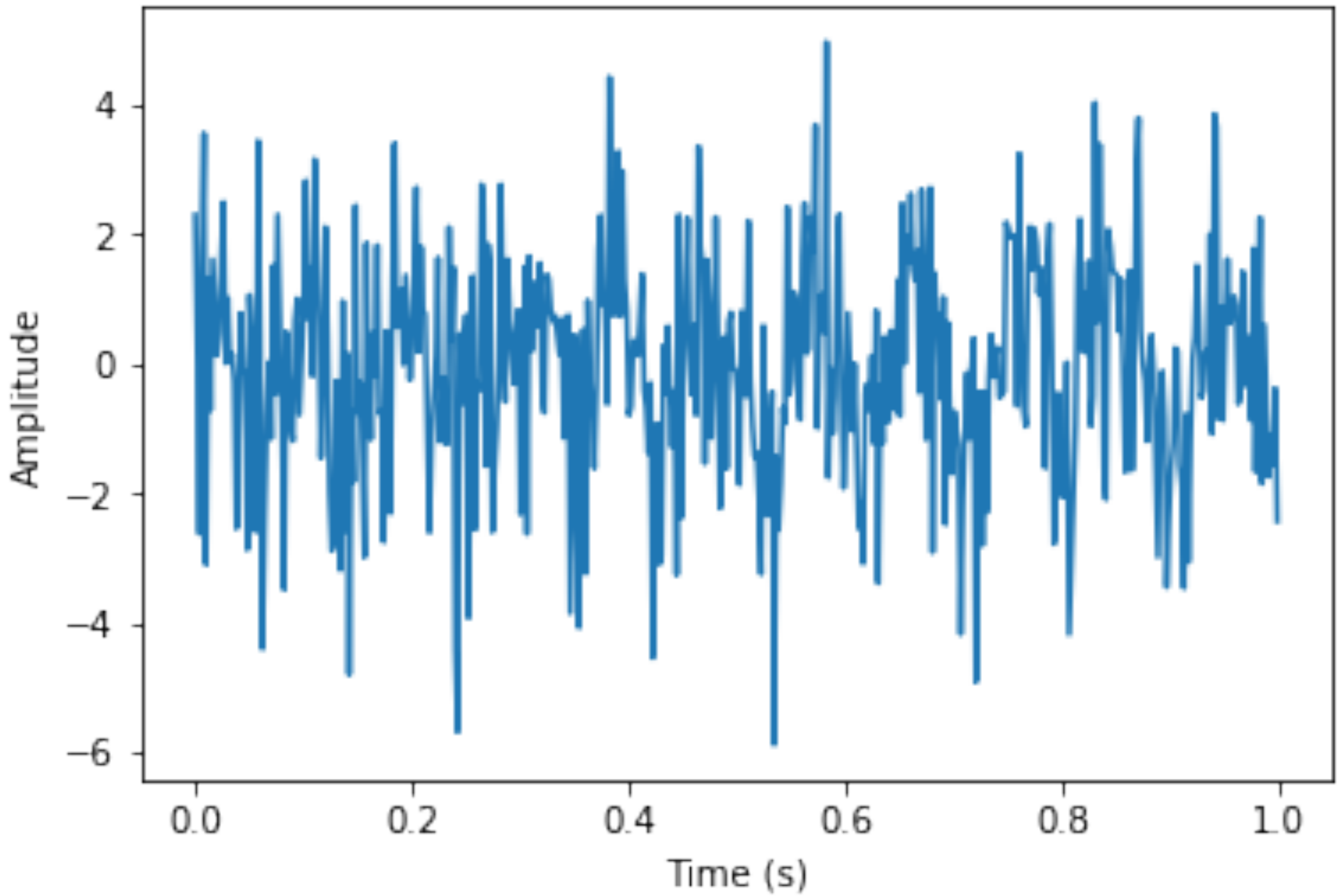


Freq. Resolution

ν_s = 50 Hz, ν_0 = 10.5 Hz, T = 3s

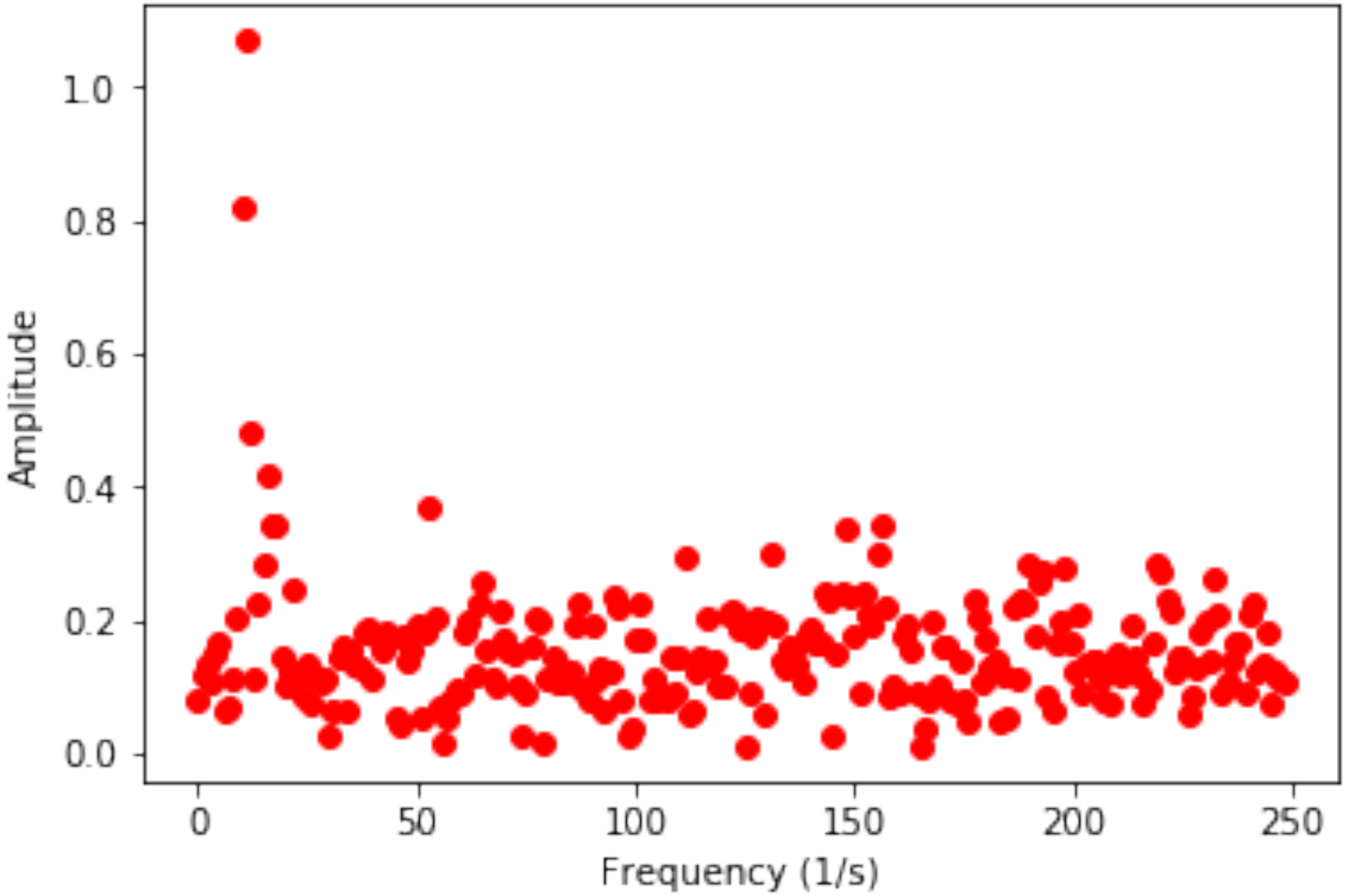


Noise



Can you find the signal in here?

Noise



How about in the Fourier Transform?