In this document you can find a list of suggested topics for undergraduate readings, together with the names of graduate student who have indicated interest in mentoring such a reading. This list is only a starting point—if you have a related, or completely different topic in mind, please discuss this with the mentors and we will try to find a mentor who suits your interests!

After reading the topics, please fill out the Interest Form that we’ve provided and turn it in. We will use the forms to assign mentors and topics in the next few weeks.

For questions, please email Christy (chazel@uoregon.edu) or Mike (mgartner@uoregon.edu). The emails of the mentors are also provided below.

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**Mentors’ Emails**

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<thead>
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Suggested Projects

Algebra-Related Projects:

• Category Theory Ivo
• Coxeter Groups Elijah
• Cryptography Sarah
• Geometric Group Theory Ivo
• Introduction to $p$-adic numbers Cathy
• Quadratic Forms Jon
• Polynomials over the Integers Elisa
• Quivers Elijah
• Representations and Characters of Groups Elijah, Ilan
• Ruler and Compass Constructions Elisa
• The Symmetry of Things Cathy

Analysis-Related Projects:

• Complex Variables Andrew, Joe
• Fourier Analysis Joe
• Statistics Andrew
• Topics in Statistics and Probability Nate

Geometry & Topology-Related Projects:

• Chaotic Dynamical Systems Eric
• Classical Algebraic Geometry Sarah, Andrew
• Classical Mechanics and Symplectic Geometry Mike
• Euler’s Formula Christy
• The Geometry of Quantum States Eric
• Knot Theory Jeff, Mike
• Riemann Surfaces Sarah
• Point Set Topology Ivo
• Quaternions Gabe
Miscellaneous Projects:

- Set Theory and Logic  
  Ilan
- Axiomatic Set Theory  
  Jeff
- Graph Theory  
  Elisa
- Game Theory  
  Gabe, Jeff
- Math History  
  Fill
- Mathematical Physics  
  Andrew, Fill
- Mathematical Problem Solving  
  Nate
- Philosophy of Mathematics  
  Fill
- $SO(3)$ and the Hydrogen Atom  
  Jay
- A Survey of Great Theorems in Math  
  Christy
- Elliptic curves  
  Sarah
**Project Abstracts**

**Algebra-Related Abstracts:**

**Category Theory** *(Ivo).* Are you a fan of abstraction? Do you think there might be a better unifying language for mathematics than set theory? Using Emily Riehl’s book ‘Category Theory in Context’ we will look at the language of category theory which has provided the foundations for many of the twentieth century’s major advances in pure mathematics. We will range widely over the mathematical landscape (although mostly avoiding analysis land) choosing and introducing examples based on your mathematical background. Time permitting we will talk about; categories, functors, natural transformations, universal properties, limits and colimits, and adjunctions. Students should have taken at least one proof based course. Your background will determine whether we spend more time talking about interesting examples of categories or more general theory (although either way we will do both).

**Coxeter Groups** *(Elijah).* Coxeter Groups are groups given by presentations of a certain form; examples include the familiar symmetric groups, dihedral groups, and many important groups arising from Lie theory. While group presentations can often be hard to work with, a rich theory of Coxeter groups has been developed using various methods. In this course we will focus on symmetric groups and learn how their Coxeter presentation sheds new light on their structure. In particular, we’ll learn about a non-trivial partial order on a symmetric groups called the Bruhat order.

**Cryptography** *(Sarah).* Cryptography uses number theory and computational complexity theory to design codes which are difficult to break. For example, an elegant application of Euler’s totient theorem, the RSA algorithm is one of the most widely used encryption schemes, but it fits on an index card! We’ll focus on the math behind various methods of encryption, using abstract algebra, number theory, and maybe even elliptic curves to encode and decode information.

**Geometric Group Theory** *(Ivo).* Groups and geometry are ubiquitous in mathematics. We will look at infinite groups from the geometrical viewpoint and will draw on ideas from low dimensional topology and from hyperbolic geometry, and combinatorics. The principal focus is the interaction of geometry/topology and group theory: through group actions and suitable translations of geometric concepts into a group theoretic setting. We will draw some pretty pictures of Cayley graphs. Topics that we might cover include; the Banach-Tarski paradox, the Fundamental group, covering spaces, decision problems, and Coxeter groups. You should have had some exposure to groups, up to at least the first isomorphism theorem.

**Introduction to \(p\)-adic numbers.** *(Cathy).* For a prime \(p\), the \(p\)-adic numbers, denoted \(\mathbb{Q}_p\), can be thought of an analogue to the real numbers in that they are a completion of the rational numbers, denoted \(\mathbb{Q}\), with respect to some metric. There are many rich and interesting properties that can be derived from the local property of \(p\)-adic numbers, and in this project, we will explore these properties through a reading of Koblitz’s *\(p\)-adic Numbers, \(p\)-adic Analysis, and Zeta-Functions*. Prereq: 315 Suggested: 341

**Quadratic Forms** *(Jon).* Symmetric and alternating bilinear forms are important in various areas of mathematics. The story for alternating forms is the same for any field, but for symmetric forms it changes drastically. We would study symmetric bilinear forms through the lens of quadratic forms, and explore constructions related to them like the Witt group. Knowledge of linear algebra and a familiarity with fields would be helpful.
Polynomials Over the Integers (Elisa). Let $p$ be a polynomial in two (or more) variables with integer coefficients. Find all integer solutions to the equation $p = 0$. This is a completely elementary question to state, but it can be incredibly difficult to answer completely, depending on the polynomial $p$. These are some of the oldest problems in mathematics, and they continue to be of interest in the present day. A few examples:

1. The integer solutions to the equation $x^2 + y^2 = z^2$ are of fundamental importance, as any Math 112 student knows.
2. There are no integer solutions to $x^3 + y^3 = z^3$, or indeed to $x^n + y^n = z^n$ for any $n \geq 3$. This is one of the most famous and difficult problems in the history of mathematics, and was only recently solved by Andrew Wiles in 1996.
3. The integer solutions to $x^2 - ny^2 = 1$ are the integer points on a hyperbola.

Considering such equations quickly leads to deep mathematics. In this project, we will begin by looking at several examples and solving easy cases. We will encounter notions of unique factorization, rings of polynomials and their ideals, and other aspects of algebraic number theory. There are many different directions this project could go, depending on the interests of the student. This project should be accessible to any student who has had some experience writing proofs. Some basic abstract algebra would be helpful but not necessary.

Quivers (Elijah). A quiver is a directed graph—a collection of dots with various arrows connecting them. From this simple mathematical object intriguing structures are born: the path algebra, a ring with elements associated to paths in the quiver, and quiver representations, which consist of vector spaces placed at each vertex and linear maps on each arrow. Quivers provide an interesting ‘hands-on’ introduction to algebras and representation theory, and pop up frequently in a number of areas of current research, from algebraic geometry to theoretical physics. Then there’s the remarkable ‘Gabriel’s Theorem’, which classifies the quivers that have a certain finiteness property, and hints at these connections to other areas of mathematics.

Representations and Characters of Groups (Elijah, Ilan). An essential tool for understanding a group $G$ is to examine how it acts on some other object. When that other object is a vector space, this means we can view group elements as matrices and the group operation as matrix multiplication. More importantly though, this type of group action, called a representation, allows us to bring to bear all the power of linear algebra on the problem of understanding groups, and results in a beautifully rich theory which has applications in all realms of mathematics and physics. In particular, when $G$ is a finite group and the vector space is complex, one finds that much of the essential information about $G$ and its representations can be condensed into a compact sudoku-like character table.

Ruler and Compass Constructions (Elisa). In around 300 BC, Euclid lays out the framework for ruler-and-compass constructions in his Elements. In such constructions, a particular set of moves are allowed using only the tools of an unmarked straight-edge and a compass in order to create a geometric object. Euclid was able to describe how to construct many objects using these moves alone. However, the limitations on these constructions remained unknown for many years. While Euclid was able to construct many regular polygons, for example, it was not until the mid 19th century that Wantzel was able to classify which regular polygons are constructible by this method. In this project, we will explore some of Euclid’s constructions, and learn some algebraic tools in order to solve what Euclid could not. Depending on interest and background, we can then learn some basics of Galois theory, and see how this can be used to further solve our classical problem. Some experience writing proofs is recommended, but this project can easily be structured to fit a variety of backgrounds.
The Symmetry of Things (Cathy). This project will be centered around John Conway’s book *The Symmetry of Things*, which focuses on understanding symmetries from both a group-theoretical point of view as well as from a more direct, hands-on approach. In order to explore both approaches, this project will have two main components: learning basic abstract algebra (mainly group theory) and exploring symmetries of solids through origami!

**Analysis-Related Abstracts:**

**Complex variables (Andrew).** Take complex numbers, add in calculus and you get complex variables. Complex numbers are already rich in mathematical beauty, but when you look at differentiable complex functions with complex variables you get an even more marvelous structure. Prereq: 252. Suggested: 253.

**Complex variables (Joe).** It is easy to find a polynomial having real coefficients and no real roots. For example, the roots of \( f(x) = x^2 + 1 \) are \( x = i \) and \( x = -i \) (where \( i := \sqrt{-1} \)), and neither of these is a real number. However, if \( p(x) \) is any polynomial with coefficients in the complex numbers \( \mathbb{C} = \{a + bi : a, b \in \mathbb{R}\} \), then every root of \( p(x) \) is also contained in \( \mathbb{C} \). This marvelous fact, known as the Fundamental Theorem of Algebra, greatly distinguishes \( \mathbb{C} \) from \( \mathbb{R} \). Loosely speaking, it implies that \( \mathbb{C} \) is “more rigid” than \( \mathbb{R} \). But how does one prove it, to what extent do we mean “more rigid”, and how can this rigidity be exploited? The answers to all of these questions lie in a strange and beautiful area of mathematics called Complex Analysis. We will begin with basic complex arithmetic and make our way towards contour integration, exploring trigonometry, power series, and many other interesting topics along the way. After choosing a particular topic, we use LaTeX to organize or figures, calculations, and explanations on a nice poster in the Spring. Tentative Goal Topics: Polar decomposition, holomorphic functions, power series representations, Cauchy’s Theorem, Liouville’s Theorem, the Residue Theorem, contour integration. Recommended prerequisites: Math 251, 252, 253, (Optional: 316, 317)

**Fourier Analysis (Joe).** It was realized in the 1700’s that many periodic functions on \( \mathbb{R} \) can be expressed in the form

\[
f(x) = \sum_{n=0}^{\infty} [a_n \cos(nx) + b_n \sin(nx)].
\]

Joseph Fourier made this idea rigorous in 1808. He used these “trigonometric series” to solve problems involving heat conduction, marking the beginning of a powerful subject in mathematics known today as Fourier analysis. Over the last 200 years, Fourier analysis has found countless applications to engineering, signal processing, differential equations, and many areas of physics. It has also been significantly generalized in the world of pure mathematics, playing major roles in complex analysis, probability theory, number theory, and harmonic analysis on locally compact abelian groups. Though the full story of Fourier analysis is deep, the basic ideas are fun, tangible, and accessible to students with various backgrounds and interests. By mixing calculus and linear algebra, we will define Fourier series and Fourier transforms. We will investigate some of their applications and properties and use graphing software to see what it all means. We will also use LaTeX to organize our figures, calculations, and explanations on a nice poster in the Spring. Tentative Goal Topics: Fourier series, Fourier transforms, the Gibbs Phenomenon, the Sampling Theorem, the Uncertainty Principle. Recommended prerequisites: Math 251, 252, 253, 341.
Statistics (Andrew). Data is beautiful! This project is for someone who wishes to learn some of the more advanced parts of statistics. We will spend time on the theory and applications of stats, and ideally we will learn some kind of statistical programming along the way.

Topics in Statistics and Probability (Nate). Probability and Statistics cover a vast array of abstract and applied topics, for the novice and advanced student alike! Some topics include:

- **Special Probability Distributions.** Why is the Normal distribution normal? What does it mean to say that the exponential distribution is memoryless? Is every distribution just a Gamma Distribution in disguise? Why does the Cauchy distribution have no average value? Why are confidence intervals for the Binomial Distribution so complicated?

- **Markov Chains.** Common wisdom says that a deck of cards must be shuffled at least 7 times in order to be truly random. Why 7? It turns out that card shuffling is an example of a Markov Chain, a process which randomly evolves over time according to a particular rule, and where the next state only depends on the current state, and nothing else.

- **The Monte Carlo Method.** One way to approximate solutions to difficult equations is to randomly sample possible solutions, evaluate an equation at each of these points, and aggregate the results to find an estimate of the true solution. For example, to estimate the area of a circle, randomly choose 1000 points \((x, y)\) with \(-1 < x < 1\), and \(-1 < y < 1\), and evaluate \(x^2 + y^2\). Then the proportion of the points with result less than 1 is approximately the ratio of the area of the circle to the area of the square of side length 2. How many points are necessary in order to ensure a ‘good’ approximation?

- **Probabilistic Number Theory.** Using methods from probability, we can answer traditional questions in the field of number theory. For example, what is the likelihood that a randomly chosen integer is prime? Once we figure out exactly what we mean by ‘randomly chosen integer,’ we can talk about the distribution of prime numbers, which is related to the well-known prime number theorem.

- **Random Matrix Theory.** Originally initiated by research in multivariate statistics, the study of random matrices is now an active area of study, with numerous applications to theoretical physics, number theory, finance, and optimization. At its heart, the field is concerned with the distribution of eigenvalues of matrices whose entries are random numbers. But it turns out that many real world phenomenon (the waiting times between trains at a train station, the positions of particles in strong magnetic fields, etc.) can be modeled by these eigenvalues! For this reading, students should be familiar with linear algebra, multivariable calculus, and probability.

**Geometry & Topology-Related Abstracts:**

**Chaotic Dynamical Systems (Eric).** A lot of the things we learn early in our mathematical lives are of the form “this seems complicated, but if we study it enough, we can see that there are some simple rules underneath all this complexity.” The subject of dynamical systems seems to be saying exactly the opposite: There are systems we can invent with very simple rules, and when we turn them loose they do wildly, frighteningly unpredictable things. I’d like to reread Robert Devaney’s book with a student and wrestle with this chaos. We might enlist computers to help us, or approach the abyss armed with only pencil and paper.
Classical Algebraic Geometry (*Sarah, Andrew*). Algebraic geometry is an old subject, which has become quite abstract in recent years. However, there is a lot of beautiful classical algebraic geometry that can be done without getting too abstract. This project will focus on varieties, which are collections of points which are zeros of a collection of polynomials. I hope to work with a student towards understanding projective spaces and projective geometry, as well as the famous theorem of Bezout.

Classical mechanics and symplectic geometry (*Mike*). Classical mechanics is the study of the motion of (often rigid) physical objects subject to forces, assuming a few governing principles or laws bearing the name of Newton. There are many interesting phenomena which occur in the study of such systems, and despite the fact that this theory of mechanics has been supplanted by another, many real world phenomena are extremely well modeled by this framework. This project could consist of learning about mechanical systems and their phase spaces, trying to answer concrete physical questions about them, and seeing what sorts of mathematical structures naturally emerge in this pursuit.

Euler’s Formula (*Christy*). Roughly speaking, a polyhedra is a 3-dimensional shape that is composed of flat polygonal faces where adjacent faces meet along a single vertical segment called an edge, and adjacent edges meet at a single corner called a vertex. Let $F$ be the number of faces, $E$ the number of edges, and $V$ the number of vertices of a polyhedra. In 1750, Euler conjectured (and “proved”) that for *any* polyhedra, $V - E + F = 2$. This simple formula has been the crux of many mathematical arguments throughout the years, many of which seem to have little to do with polyhedra. In this reading, I’d like to explore why this formula is true, and moreso, why such a simple formula has remained important in mathematics for so long.

Geometry of Quantum States (*Eric*). In chasing down a reference for my thesis, I checked out a book on quantum mechanics where I once found a complicated mathematical idea explained in a really pretty, intuitive way. I’ve been thumbing through it ever since, and while almost none of it is related to my research, I really like how it’s written. Ideas include high-dimensional geometry, information theory, color theory, calculus, group theory, and topology. WARNING: I know essentially nothing about quantum mechanics. This would be a reading where a student and I both read a section before meeting, and then we’d sit down and say “what the heck did that mean?” But the book seems well written, so I think it might be fun. And if it isn’t, we’ll decide which of the ideas we liked best and explore that instead.

Knot Theory (*Jeff, Mike*). Knot Theory is the study of mathematical knots, which can be described as follows. First, take a piece of string or rope. Tie a knot in it. Now, glue or tape the ends together. You have created a mathematical knot. The central problem of Knot Theory is determining whether two knots can be rearranged (without cutting) to be exactly alike.

Riemann surfaces (*Sarah*). Riemann surfaces are two-dimensional surfaces with a special structure, one that makes them look locally like the complex numbers. These surfaces have a both a rich analytic theory and algebraic theory, with many connections between the two. This project can be taken in many directions. Depending on the student, we will discuss manifolds and complex numbers, and develop the machinery to talk about Riemann surfaces. One could then study either the analytic side of the story or the algebraic side, depending on one’s interest and background.
**Point-Set Topology** (*Ivo*). Topology is one of the newer branches of mathematics, originating at about the close of the nineteenth century and beginning of the twentieth. The word topology only came to be commonly used later. The first tracts on topology included Poincare’s Analysis situs (Analysis of Place) and Hausdor’s Mengenlehre (Set Theory). The twentieth century saw a rapid expansion in the study of topology. Several branches of topology emerged as disciplines in their own right, including point-set topology, algebraic topology, differential topology and geometric topology. Category theory and homological algebra both grew out of (algebraic) topology. Hardly any branch of mathematics remains untouched by topology. One way to regard topology is as the attempt to understand continuity in its broadest possible context. This is the perspective we adopt. We will first discuss metric spaces, the is sets with a concept of distance. From there we will generalise to topological spaces. Topics may include; connectedness, separation properties, compactness, homotopy, and compactification. Time permitting we might look at the weird and wonderful book ‘Counterexamples in Topology’. Students should have taken at least one proof based course.

**Quaternions** (*Gabe*). You’ve heard of complex numbers built out of a real and imaginary part: \( z = x + iy \). With complex numbers these parts can each be represented by a real number: \( x \) and \( y \). Quaternions take the imaginary part and instead of allowing just a number, it allows a whole 3-dimensional imaginary vector. Quaternions are fundamental for many applications from 3D computer graphics, to physics. We will learn how to understand and visualize quaternions, what their properties are, and how to use them. Variable level: what we cover can be tailored to what you already know.

**Miscellaneous Abstracts:**

**Set Theory and Logic** (*Ilan*). Set theory and Mathematical Logic are the basis of (almost) all mathematics. At its core, set theory is about (finite or infinite) collections of objects. Mathematical Logic is hard to describe, but fundamental to mathematics. It deals with the study of many odd properties in mathematics, such as provability, computability, decidability (i.e. given a set and an object, do we have a way of deciding whether or not that object is in that set?). This project could start with the basics of set theory, and then could branch in any of a number of directions. A more set theoretic topic could include studying different infinities, or exploring the axiom of choice. A more logic-oriented topic could be sentential logic, first order logic - a very ambitious project might even aim to understand one or both of Godel’s incompleteness theorems (although proving this would be rather difficult). Any of these topics has a slew of applications to mathematics, as well as computer science.

**Axiomatic Set Theory** (*Jeff*). Set theory is a branch of mathematics started in 1874 by mathematician Georg Cantor, and has since become the basis for most of modern mathematics. From the simple idea of defining a set as a (possibly infinite) collection of objects, a huge world of interesting and sometimes unbelievable math emerges. This world inculdes exploring why “Infinity plus 1” actually makes sense, confronting the “paradoxes” that arise from the axiom of choice, and even possibly learning how to defeat a monster with an infinite number of heads. This project will start with the basics of set theory and so is great for those with little to no background in advanced mathematics.

**Graph Theory** (*Elisa*). Graphs are just collections of points connected by collections of lines, but this simple structure can be used to study many kinds of relationships, and gives rise to some interesting problems in probability, combinatorics, computer science and data analysis. The subject has many simply-stated questions that remain unsolved.
**Game Theory** *(Gabe)*. At the intersection of economics, psychology, political science, and logic lies Game Theory: a mathematical study that uses tools from linear algebra, calculus, and probability to answer fundamental questions about strategy. In this reading we will explore Game Theory: an Introduction by E.M. Barron. We will learn how to represent simple cooperative and competitive games with matrices, and how to use mathematical tools to analyze which strategies end up being most successful. 251, 252, 341 suggested.

**Game Theory** *(Jeff)*. Game theory refers to a branch of mathematics that uses math to try to analyze the strategy and decision making in various types of games. Whether we use logic to find a guaranteed win in games like Nim, use probability to get an advantage in Rock-Paper-Scissors, or use modular arithmetic to increase our chance of success at Hanabi, Game theory is all about finding the move that gives you the best chance of victory. Depending on people’s interest and backgrounds, this reading can vary from concrete mathematical proofs to more general concepts involving strategy and tactics. Either way expect to spend some time having fun playing a bunch of games.

**Math History** *(Fill)*. One issue that makes mathematics difficult to learn is that the order in which we learn concepts is not the order in which they were discovered! For instance a rigorous notion of the set of ‘real numbers’ did not exist until about 125 years ago. Understanding the history of mathematics can help one understand the notation and language that is used as well as give light to the motivation for developing these areas of mathematics. The exact subject and time period of study is left up to the reader as there are hundreds of interesting moments in the history of mathematics.

**Mathematical Physics** *(Andrew, Fill)*. There is a huge interplay between mathematics and physics at all levels. This project can take on a myriad of different flavors, mainly determined by the interest of the student. Possibilities include (but are not limited to) group representation theory in quantum mechanics; differential geometry in classical mechanics/general relativity/electromagnetism; and functional analysis on Hilbert spaces as the underpinnings of quantum mechanics.

**Mathematical Problem Solving** *(Nate)*. (Putnam 1990) Consider a hole puncher that can be centered at any point in the plane, and that, when operated, removes from the plane precisely those points whose distance from the center is irrational. How many punches are needed to remove every point?

Most homework in math classes consist of *exercises*: tasks which may be hard or easy, but are rarely puzzling. The solutions to exercises may involve tricky technical work, but the path forward is almost always apparent. In contrast, *problems* are questions that cannot be answered immediately, which are often open-ended, paradoxical, and sometimes even unsolvable. Problems are adventures, encouraging travelers to get lost along the way. Like programming, cooking, or painting, mathematical problem solving is a skill that can be learned and further refined through practice. Depending on a student’s background, this reading could apply problem solving strategies to a variety of topics: algebra, combinatorics, number theory, calculus, geometry. Although not a requirement, this reading may be especially interest students taking the Putnam Exam in December.

**Philosophy of Mathematics** *(Fill)*. This subject deals with philosophical questions about mathematics. There are questions of metaphysics: Does mathematics have a subject-matter? Then there are semantic matters: What is the nature of mathematical truth? And epistemology: How is mathematics known? What is a proof?
**SO(3) and the Hydrogen Atom** (Jay). As new mathematical objects are invented, they will naturally carry symmetries in the form of their automorphisms. Oftentimes understanding these symmetries in their own right resolves many things about the original mathematical object. This project will explore the role symmetry plays in physics. A recurring philosophy in physics is Noether’s theorem, which posits that continuous symmetries of physical equations correspond to conserved quantities of a physical system. Time symmetry leads to conservation of energy, translational symmetry leads to conservation of momentum, and rotational symmetry leads to conservation of angular momentum. In the quantum world, Noether’s theorem is actually easier to understand and prove, and is beautifully exhibited by the spherical symmetry of Hydrogen-like atoms which lead to conservation of quantum angular momentum. The momenta of the Hydrogen atom are governed by the system of symmetries known as SO(3), and studying how these symmetries can appear in nature, e.g. as states of the hydrogen atom e.g. its representations, explains the organization of the periodic table.

The punchline of this project is that everyone secretly studied the representation theory of SO(3) back in their high school chemistry classes!

This project is best suited for someone with an interest in physics and some background in linear algebra and differential equations. We will use appropriate references based on the mentee’s prior background in math and physics.

**A Survey of Great Theorems in Math** (Christy). This project will be centered around William Dunham’s book *Journey through Genius*. The following quotation is from the preface of his book.

> For disciplines as diverse as literature, music, and art, there is a tradition of examining masterpieces-the ‘great novels,’ the ‘great symphonies’, the ‘great paintings’-as the fittest and most illuminating objects of study. Books are written and courses are taught on precisely these topics in order to acquaint us with some of the creative milestones of the discipline and with the men and women who produced them. The present book offers an analogous approach to mathematics, where the creative unit is not the novel or symphony, but the theorem.

In this reading, I’d like to explore different chapters in Dunham’s book. Each chapter focuses on one “great theorem”. This would be a great project for someone who is just getting into more advanced math and who would like to learn more about what pure mathematics is all about.

**Elliptic curves** (Sarah). Elliptic curves are smooth curves defined as the points which are zeros of a degree three polynomial. This makes them a particularly nice first example of an algebraic variety, but they come with even more structure: their points form an algebraic group. This has allowed mathematicians to study them from many different angles, and they’ve turned out to be very important in number theory and cryptography. For this project, we will work out of the book “Rational Points on Elliptic Curves,” and after the basics we can move in lots of different directions depending on the student’s interests.
AWM Undergraduate Reading Program: Interest Form

Please fill this form and turn it in. If you’d like to turn it in later, please return it to Mike Gartner or Christy Hazel’s mailbox (behind the Math Office in Fenton) by Monday, November 13th.

Your name: ________________________________________________

Your email: ________________________________________________

Year in school: _____________________________________________

Circle the math classes you’ve taken or are taking:

- 111  - 211  - 231  - 246  - 251  - 252  - 253
- 112  - 212  - 232  - 247  - 252  - 253
- 211  - 212  - 233  - 246  - 251  - 252
- 213  - 213  - 233  - 247  - 251  - 253
- 231  - 232  - 233  - 246  - 251  - 253
- 261  - 262  - 263  - 281  - 307  - 315
- 256  - 256  - 282  - 307  - 315  - 341
- 261  - 256  - 341  - 342  - 391  - 392
- 262  - 256  - 341  - 342  - 391  - 392
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Any other courses (list them here):
Below, list your top five choices for reading topic:

1. 
2. 
3. 
4. 
5. 

Below, list your top five choices for reading advisor:

1. 
2. 
3. 
4. 
5. 

Here are the names of advisors:

Jon Aycock  
Elisa Bellah  
Elijah Bodish  
Sarah Frei  
Mike Gartner  
Jay Hathaway  
Christy Hazel  
Paul Herstedt  
Eric Hogle  
Cathy Hsu  
Gabe Montes de Oca  
Jeffrey Musyt  
Fil Staley  
Ivo Vekemans  
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