Math 246 (9-10am), Quiz 1.

0. Write your name here:

1. Find the limit: $\lim_{x\to 2} \frac{2}{3x+4}$ and justify your answer.

Function $\frac{2}{3x+4}$ is defined at $x=2$, so it is continuous at $x=2$. Hence

$$\lim_{x\to 2} \frac{2}{3x+4} = \frac{2}{3\cdot 2+4} = \frac{2}{10} = .2.$$

Answer: $\lim_{x\to 2} \frac{2}{3x+4} = .2$

2. Referring to Problem 1, how close the input must be to 2 for the output to be within 0.01 of the limit?

We need to solve the following inequalities:

$$.19 \le \frac{2}{3x+4} \le .21$$

$$\frac{1}{.19} \ge \frac{3x+4}{2} \ge \frac{1}{.21}$$

$$\frac{2}{.19} \ge 3x+4 \ge \frac{2}{.21}$$

$$\frac{1}{3}\left(\frac{2}{.19}-4\right) \ge x \ge \frac{1}{3}\left(\frac{2}{.21}-4\right)$$

$$2.175 \ge x \ge 1.841$$

Answer: the input should be between 1.841 and 2.175

3. Set up the limit computing the slope of the tangent line to the graph of the function $f(x) = 3x^2$ with base point $x = 2$. Simplify this expression and compute the limit.

The limit computing the slope is

$$f'(2) = \lim_{\Delta x\to 0} \frac{3(2+\Delta x)^2 - 3\cdot 2^2}{\Delta x} = \lim_{\Delta x\to 0} \frac{3(4+4\Delta x+\Delta x^2)-12}{\Delta x} =$$

$$= \lim_{\Delta x\to 0} \frac{12+12\Delta x+3\Delta x^2-12}{\Delta x} = \lim_{\Delta x\to 0} \frac{\Delta x(12+3\Delta x)}{\Delta x} =$$

$$= \lim_{\Delta x\to 0} (12+3\Delta x) = 12$$

Answer: the slope of the tangent line is 12.