Math 246 (9-10am), Quiz 2.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Find the derivative of $f(x) = x^2(3\sqrt{x} - 1)$.

We use Product Rule and $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$:

$$f'(x) = 2x(3\sqrt{x} - 1) + x^2 \cdot 3\frac{1}{2\sqrt{x}} = 6x\sqrt{x} - 2x + \frac{3}{2}x\sqrt{x} =$$

$$= \frac{15}{2}x\sqrt{x} - 2x.$$

Answer: $f'(x) = \frac{15}{2}x\sqrt{x} - 2x$.

2. The size of some population at moment of time t is given by $N(t) = \frac{500t}{3+\sqrt{t}}$. What is instantaneous rate of change of the size of population at the moment $t = 4$?

We need to compute $N'(4)$. We use Quotient Rule and $(\sqrt{t})' = \frac{1}{2\sqrt{t}}$

$$N'(t) = \frac{500(3+\sqrt{t}) - 500t \cdot \frac{1}{2\sqrt{t}}}{(3+\sqrt{t})^2} = \frac{1500 + 500\sqrt{t} - 250\sqrt{t}}{(3+\sqrt{t})^2} = \frac{1500 + 250\sqrt{t}}{(3+\sqrt{t})^2}$$

Hence $N'(4) = \frac{1500 + 250\sqrt{4}}{(3+\sqrt{4})^2} = \frac{1500 + 500}{5^2} = \frac{2000}{25} = 80$

Answer: the instanteneous rate of change of the size of population at $t = 4$ equals 80

3. Find the slope of the tangent line to the graph of the function $g(x) = \sqrt{2x^2 + 7}$ with base point $x = 3$.

We are asked to compute $g'(3)$. We use Chain Rule and $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$:

$$g'(x) = \frac{1}{2\sqrt{2x^2+7}} \cdot (2x^2+7)' = \frac{4x}{2\sqrt{2x^2+7}} = \frac{2x}{\sqrt{2x^2+7}}$$

Thus $g'(3) = \frac{2 \cdot 3}{\sqrt{2 \cdot 3^2 + 7}} = \frac{6}{\sqrt{18+7}} = \frac{6}{\sqrt{25}} = \frac{6}{5} = 1.2$

Answer: the slope of the tangent line to the graph of $g(x)$ with base point $x = 3$ is $\frac{6}{5} = 1.2$.