Math 246 (9-10am), Quiz 3.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Consider the discrete time dynamical system $x_{t+1} = \frac{2x_t}{3x_t+2}$ with initial condition $x_0 = 1$. Compute x_1, x_2, x_3, x_4.

$X_1 = \frac{2\cdot 1}{3\cdot 1+2} = \frac{2}{5}$, $X_2 = \frac{2\cdot\frac{2}{5}}{3\cdot\frac{2}{5}+2} = \frac{2\cdot\frac{2}{5}\cdot 5}{(3\cdot\frac{2}{5}+2)\cdot 5} = \frac{4}{3\cdot 2+5\cdot 2} = \frac{4}{16} = \frac{1}{4}$

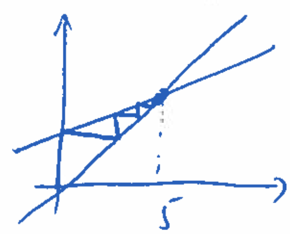
$X_3 = \frac{2\cdot\frac{1}{4}}{3\cdot\frac{1}{4}+2} = \frac{2}{3+2\cdot 4} = \frac{2}{11}$, $X_4 = \frac{2\cdot\frac{2}{11}}{3\cdot\frac{2}{11}+2} = \frac{4}{3\cdot 2+2\cdot 11} = \frac{4}{28} = \frac{1}{7}$

Answer: $x_1 = \frac{2}{5}$, $x_2 = \frac{1}{4}$, $x_3 = \frac{2}{11}$, $x_4 = \frac{1}{7}$ (there is a pattern here!)

2. Assume that the concentration of a medication in the bloodstream is described by the dynamical system $M_{t+1} = .4M_t + 3$ with initial condition $M_0 = 0$. What is M_{100} approximately? Explain your guess!

The updating function is $f(x) = .4x+3$, so $f'(x) = .4 < 1$, so any fixed point is stable. Thus it seems reasonable to expect that in a long run M_t is very close to the equilibrium. Let us find it: $x = .4x+3$, $.6x = 3$, $x = 3/.6 = 5$.

Answer: we expect $M_{100} \approx 5$. Here is cobwebbing picture:



3. Consider the discrete time dynamical system $a_{t+1} = \frac{2}{3}a_t(1+a_t)$. Find the fixed points and determine their stability.

Updating function $f(x) = \frac{2}{3}x(1+x)$

equilibrium equation $x = \frac{2}{3}x(1+x)$, $x=0$ or $1 = \frac{2}{3}(1+x)$

$\frac{3}{2} = 1+x$

$x = \frac{1}{2}$

Stability: $f'(x) = \left(\frac{2}{3}(x+x^2)\right)' = \frac{2}{3}(1+2x)$

$f'(0) = \frac{2}{3} < 1$, so $x=0$ is stable

$f'(\frac{1}{2}) = \frac{2}{3}(1+2\cdot\frac{1}{2}) = \frac{2}{3}(1+1) = \frac{4}{3} > 1$, so $x = \frac{1}{2}$ is unstable

Answer: there are two fixed points: stable $x=0$ and unstable $x = \frac{1}{2}$