Math 246, Review problems for Midterm II.

1. Consider a discrete time dynamical system $b_{t+1} = b_t - 2$ with the initial condition $b_0 = 500$. Write a closed-form expression for b_t.

Answer: $b_t = 500 - 2t$.

2. Consider a discrete time dynamical system $M_{t+1} = 2\sqrt{M_t} + 3$ with the initial condition $M_0 = 3$. What is M_{1000} approximately?

Answer: $M_{1000} \approx 9$ (via cobwebbing).

3. Find the fixed points and determine their stability for the dynamical system $a_{t+1} = a_t^2 - 1$.

Answer: fixed points are $\frac{1 \pm \sqrt{5}}{2}$, both unstable.

4. Find stable fixed points for the dynamical system $N_{t+1} = \frac{2N_t}{2N_t + 1}$.

Answer: the only stable fixed point is $N = \frac{1}{2}$.

5. Find the global maximum and minimum of the function $f(x) = x - \sqrt{x}$ on the interval $[0, 1]$.

Answer: the global maximum is 0, attained at $x = 0$ and $x = 1$; the global minimum is $-\frac{1}{4}$, attained at $x = \frac{1}{4}$.

6. Find the local maxima and minima of the function $f(x) = x^3 - 3x + 2$.

Answer: there are two critical points $x = -1$ and $x = 1$; $x = -1$ is local maximum and $x = 1$ is local minimum.

7. What is global maximum of the function $f(t) = t^2(1-t)^3$ on the interval $[0, 1]$?

Answer: the global maximum is $\frac{108}{3125}$, attained at $t = \frac{2}{5}$.

8. Find global extrema of the function $f(x) = x^3 - x^2$ on the interval $[-1, 1]$.

Answer: the global maximum is 0, attained at $x = 0$ and $x = 1$; the global minimum is -2, attained at $x = -1$.

9. Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

Answer: $9 = 3 + 6$ and the maximum product is $3 \cdot 6^2 = 108$.

10. An open rectangular box with square base is to be made from 48 square feet of material. What dimensions will result in a box with the largest possible volume ?

Answer: the optimal size is $4 \times 4 \times 2$.