# Math 246 (11-12am), Quiz 1.

0. Write your name here:

1. Find the limit: $\lim_{x\to 3}\frac{6}{4x+3}$ and justify your answer.

The function $\frac{6}{4x+3}$ is defined at $x=3$, so it is continuous at $x=3$. Hence

$$\lim_{x\to 3}\frac{6}{4x+3}=\frac{6}{4\cdot 3+3}=\frac{6}{15}=.4$$

Answer: $\lim_{x\to 3}\frac{6}{4x+3}=.4$

2. Referring to Problem 1, how close the input must be to 3 for the output to be within 0.01 of the limit?

We need to solve the following inequality:

$$.39\le\frac{6}{4x+3}\le .41$$

$$\frac{1}{.39}\ge\frac{4x+3}{6}\ge\frac{1}{.41}$$

$$\frac{6}{.39}\ge 4x+3\ge\frac{6}{.41}$$

$$\frac{1}{4}\left(\frac{6}{.39}-3\right)\ge x\ge\frac{1}{4}\left(\frac{6}{.41}-3\right)$$

$$3.10\ge x\ge 2.91$$

Answer: the input must be between 2.91 and 3.10

3. Set up the limit computing the slope of the tangent line to the graph of the function $f(x)=2x^2$ with base point $x=3$. Simplify this expression and compute the limit.

The slope of the tangent line is $f'(3)$, which is given by the following limit:

$$f'(3)=\lim_{\Delta x\to 0}\frac{2(3+\Delta x)^2-2\cdot 3^2}{\Delta x}=\lim_{\Delta x\to 0}\frac{2(9+6\Delta x+\Delta x^2)-18}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}\frac{18+12\Delta x+2\Delta x^2-18}{\Delta x}=\lim_{\Delta x\to 0}\frac{12\Delta x+2\Delta x^2}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}(12+2\Delta x)=12$$

Answer: the slope of the tangent line is 12.