Math 246 (11-12am), Quiz 3.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Consider the discrete time dynamical system $x_{t+1} = \frac{2x_t}{3x_t+2}$ with initial condition $x_0 = 2$. Compute x_1, x_2, x_3, x_4.

$x_1 = \frac{2\cdot 2}{3\cdot 2+2} = \frac{4}{8} = \frac{1}{2}$; $x_2 = \frac{2\cdot\frac{1}{2}}{3\cdot\frac{1}{2}+2} = \frac{1}{\frac{7}{2}} = \frac{2}{7}$

$x_3 = \frac{2\cdot\frac{2}{7}}{3\cdot\frac{2}{7}+2} = \frac{4}{3\cdot 2+2\cdot 7} = \frac{4}{20} = \frac{1}{5}$; $x_4 = \frac{2\cdot\frac{1}{5}}{3\cdot\frac{1}{5}+2} = \frac{2}{3+2\cdot 5} = \frac{2}{13}$

Answer: $x_1 = \frac{1}{2}$, $x_2 = \frac{2}{7}$, $x_3 = \frac{1}{5}$, $x_4 = \frac{2}{13}$ (there is a pattern here!)

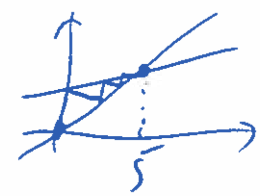
2. Assume that the concentration of a medication in the bloodstream is described by the dynamical system $M_{t+1} = .6M_t + 2$ with initial condition $M_0 = 0$. What is M_{100} approximately? Explain your guess!

The updating function here is $f(x) = .6x+2$, so $f'(x) = .6$, so any fixed point is stable.

Thus it seems reasonable to expect that in a long run M_t is very close to the equilibrium.

We find it: $x = .6x+2$, $.4x = 2$, $x = 2/.4 = 5$.

Answer: we expect $M_{100} \approx 5$ (here is cobwebbing picture:



3. Consider the discrete time dynamical system $a_{t+1} = \frac{3}{2}a_t(1+a_t)$. Find the fixed points and determine their stability.

Updating function $f(x) = \frac{3}{2}x(1+x)$

Equilibrium equation $x = \frac{3}{2}x(1+x)$ $x=0$ or

$1 = \frac{3}{2}(1+x)$

$\frac{2}{3} = 1+x$

$x = -\frac{1}{3}$

Stability: $f(x) = \frac{3}{2}(x+x^2)$, so

$f'(x) = \frac{3}{2}(1+2x)$

$f'(0) = \frac{3}{2} > 1$, so $x=0$ unstable

$f'(-\frac{1}{3}) = \frac{3}{2}(1+2(-\frac{1}{3})) = \frac{3}{2}(1-\frac{2}{3}) = \frac{1}{2} < 1$, so $x = -\frac{1}{3}$ stable

Answer: there are two fixed points; $x=0$ unstable and $x = -\frac{1}{3}$ stable.