Math 246 (11-12am), Quiz 4.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Find the local maxima and minima of the function

$$f(x) = x^3 - 3x^2 - 9x + 7.$$

We compute $f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x+1)(x-3)$

Thus the critical points are $x = -1$ and $x = 3$.

We compute $f''(x) = 6x - 6$ and $f''(-1) = -12$, $f''(3) = 12$

Thus $x = -1$ is a local maximum and $x = 3$ is a local minimum.

Answer: $x = -1$ is a local maximum, $x = 3$ is a local minimum

2. Find global extrema of the function $f(x) = x^3 - x^2$ on the interval $[0, 2]$.

1) We compute f at endpoints: $f(0) = 0$, $f(2) = 2^3 - 2^2 = 4$

2) We compute $f'(x) = 3x^2 - 2x = x(3x - 2)$ and find the critical points: $x = 0$ and $x = 2/3$.

3) We compute f at the critical points: $f(0) = 0$,

$f(2/3) = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^2\left(\frac{2}{3} - 1\right) = \frac{4}{9}\left(\frac{-1}{3}\right) = -\frac{4}{27}$

4) Answer: the global maximum is 4, attained at $x = 2$

the global minimum is $-\frac{4}{27}$, attained at $x = 2/3$

3. What is largest value attained by the function $f(x) = x^2 e^{-x}$ for positive values of x?

We compute $f'(x) = 2x e^{-x} + x^2 \cdot (e^{-x} \cdot (-1)) = e^{-x}(2x - x^2) = e^{-x}x(2 - x)$

Thus the critical points are $x = 0$ and $x = 2$.

Note that $f'(x) > 0$ for $x \in (0, 2)$ and $f'(x) < 0$ for $x \in (2, \infty)$, so $x = 2$ is the point of global maximum for positive x. Thus the largest value is $f(2) = 2^2 e^{-2} = \frac{4}{e^2}$.

Answer: the largest value is $\frac{4}{e^2}$, attained at $x = 2$.