Math 246 (11-12am), Quiz 5.

Show all your work! Use the most efficient method you know!

0. Write your name here:

1. Find $\lim_{x\to\infty} \frac{3x^2-4x+6\ln(x)}{75x-7x^2+32}$.

It seems that the leading term of both numerator and denominator is x^2. So

$$\lim_{x\to\infty} \frac{3x^2-4x+6\ln(x)}{75x-7x^2+32} = \lim_{x\to\infty} \frac{(3x^2-4x+6\ln x)\frac{1}{x^2}}{(75x-7x^2+32)\frac{1}{x^2}} = \lim_{x\to\infty} \frac{3-\frac{4}{x}+6\frac{\ln x}{x}}{\frac{75}{x}-7+\frac{32}{x^2}} = -\frac{3}{7}$$

(we used that $\ln(x)$ grows slower than x).

Answer: the limit is $-\frac{3}{7}$.

2. Find $\lim_{x\to 1} \frac{x^5+3x-4}{x^6-2x^4+1}$.

This is an indeterminate form of type $\frac{0}{0}$, so we apply L'Hôpital's Rule:

$$\lim_{x\to 1} \frac{x^5+3x-4}{x^6-2x^4+1} = \lim_{x\to 1} \frac{5x^4+3}{6x^5-2\cdot 4x^3} = \frac{8}{6-8} = \frac{8}{-2} = -4$$

Answer: the limit is -4.

3. Which of the following functions approaches zero faster as x approaches infinity: $e^{-2x}x^{-3}$ or $e^{-3x}x^{-2}$?

Let us find the limit of the ratio:

$$\lim_{x\to\infty} \frac{e^{-2x}x^{-3}}{e^{-3x}x^{-2}} = \lim_{x\to\infty} \frac{e^{-2x}x^{-3}\cdot x^3\cdot e^{3x}}{e^{-3x}x^{-2}\cdot x^3\cdot e^{3x}} = \lim_{x\to\infty} \frac{e^x}{x} = \infty$$

since e^x grows faster than any power of x. This means that the denominator $e^{-3x}x^{-2}$ is much smaller than numerator for large values of x, that is it approaches 0 faster.

Answer: $e^{-3x}x^{-2}$ approaches 0 faster than $e^{-2x}x^{-3}$ as x approaches infinity.