Math 246, Review problems for Midterm I.

1. Find the limit $\lim_{x\to 3}\frac{x^2}{6-x^2}$. How close the input must be to 3 for the output to be within 0.01 of the limit?

Answer: $\lim_{x\to 3}\frac{x^2}{6-x^2} = -3$. The input must be in the interval $(2.99751, 3.00251)$ for the output to be within 0.01 of the limit.

2. Set up the limit computing the instantaneous rate of change of the quantity $b(t) = \frac{t}{2+3t}$ at the moment $t = 2$. Simplify this expression and compute the limit.

Answer:

$$\lim_{\Delta t\to 0}\frac{\frac{2+\Delta t}{2+3(2+\Delta t)} - \frac{2}{2+3\cdot 2}}{\Delta t} = \frac{1}{32}.$$

3. Find the derivative of $f(t) = t^3(3 - 2\sqrt[5]{t})$. What are the critical points of this function?

Answer: $f'(t) = 9t^2 - \frac{32}{5}t^{11/5}$. The critical points are $t = 0$ and $t = \left(\frac{45}{32}\right)^5 \approx 5.499$.

4. Find the derivative of $f(x) = \frac{x}{\sqrt{x^3+a^2}}$ assuming that a is a constant.

Answer: $f'(x) = \frac{2a^2-x^3}{2(x^3+a^2)^{3/2}}$.

5. Find the derivative of $f(x) = e^{0.3x}\sin(2.7x + 3.5)$.

Answer: $f'(x) = 0.3e^{0.3x}\sin(2.7x + 3.5) + 2.7e^{0.3x}\cos(2.7x + 3.5)$.

6. Find the second derivative of $f(x) = \ln(x^2 + 1)$. For which values of x the graph of $f(x)$ is concave up or down?

Answer: $f''(x) = \frac{2(1-x^2)}{(x^2+1)^2}$. The graph is concave up for $x \in (-1, 1)$ and concave down for $x \in (-\infty, -1)$ and $x \in (1, \infty)$.

7. Find the tangent line to $y = \ln(x + 1)$ at $x = 1$.

Answer: $y - \ln(2) = \frac{1}{2}(x - 1)$.

8. Assume that $\ln(y - x) = 2y$. Find $\frac{dy}{dx}$ by implicit differentiation.

Answer: $\frac{dy}{dx} = \frac{1}{1-2y+2x}$.

9. Assume that $x^3 + y^3 = 9$. Find $\frac{dy}{dt}$ when $x = 1$ and $\frac{dx}{dt} = -2$.

Answer: $\frac{dy}{dt} = \frac{1}{2}$.

10. Find the tangent line to the curve $x\ln(y) = 2y\ln(x)$ at $x = 1$.

Answer: $y = 2x - 1$.

11. The concentration of a chemical in a lake at the moment of time t (time is measured in days) is given by $C(t) = \frac{t+1}{100(2+\cos(\pi t))}$. What is the rate of change of the concentration at the moment $t = 10$?

Answer: the rate of change of the concentration at the moment $t = 10$ is $C'(10) = \frac{1}{300}$.

12. The position of a particle moving on the line is $s(t) = t^3 - 6t^2 + 9t - 4$ (time is positive and is measured in seconds; the position is measured in meters). What is velocity of the particle at $t = 2$? When the particle moves to the left and when it moves to the right?

Answer: the velocity at $t = 2$ is $s'(2) = -3$ m/s. The particle moves to the right when $t \in [0, 1]$ and $t \in [3, \infty)$; it moves to the left when $t \in [1, 3]$.