TOPOLOGICAL FIELD THEORIES AND TENSOR CATEGORIES. HOMEWORK #1.

Target day for this homework: October 14

Submit (in class or by email) any of the following problems.

1. Let us work in the category Bord_2 (unoriented cobordisms). Let $\mathbf{1}$ be the unit object (i.e. empty 1-manifold) and let X be the circle S^1. Recall that we have morphisms $\alpha : \mathbf{1} \to X \otimes X$, $\beta : X \otimes X \to \mathbf{1}$, and $c_h : X \to X$ where h is an element of the mapping class group of S^1 (which is isomorphic to $\mathbb{Z}/2$). Which 2-manifold is represented by composition

$$\mathbf{1} \xrightarrow{\alpha} X \otimes X \xrightarrow{\mathrm{id}\otimes c_h} X \otimes X \xrightarrow{\beta} \mathbf{1}$$

for two possible choices of h?

2. (a) Prove that in dimension 1, the Euler theory is isomorphic to the trivial theory.

(b) Prove that in dimension 2, the Euler theory with $u \neq \pm 1$ is not isomorphic to the trivial theory.

(c)* True/False: in dimension 3, the Euler theory (say with $u = 2$) is isomorphic to the trivial theory.

3. Recall pointed tensor categories Vec_G^ω. Prove that there exists a surjective tensor functor $\mathrm{Vec}_{C_4} \to \mathrm{Vec}_{C_2}^\omega$ where ω is nontrivial.

4. Compute explicitly the associativity constraint in the Fibonacci category.

5. (a) Let us consider the following commutative diagram of finite sets:

$$\begin{array}{ccc} M & \xrightarrow{\tilde{t}} & X \\ \downarrow{\scriptstyle \tilde{s}} & & \downarrow{\scriptstyle s} \\ Y & \xrightarrow{t} & S \end{array}$$

We have two maps from functions (say real valued) on X to functions on Y: $t^* \circ s_*$ and $\tilde{s}_* \circ \tilde{t}^*$ (where t^* and $\tilde{t}^*$ are just compositions and s_* and $\tilde{s}_*$ are "integration over the fibers"). Under which conditions these two maps coincide? ("pullback" might be a useful keyword).

(b)* Replace above the finite sets by finite groupoids. The paper on "groupoidification" might be useful.