# TOPOLOGICAL FIELD THEORIES AND TENSOR CATEGORIES. HOMEWORK #2.

## Target day for this homework: October 28

Submit (in class or by email) any of the following problems.

1. Let $F : \mathcal{C} \to \mathcal{D}$ be a tensor functor and let $X \in \mathcal{C}$ be a dualizable object. Write a complete proof of the identity $F(X^*) = F(X)^*$.

2. Give an example of morphism of monoidal functors which is not an isomorphism.

3. (a) Let $G$ be a finite group and let $\mathrm{Rep}(G)$ be the category of finite dimensional representations of $G$ (say over the field of complex numbers). Let $F : \mathrm{Rep}(G) \to \mathrm{Vec}$ be the forgetful functor. Compute the group $\mathrm{Aut}^{\otimes}(F)$ of tensor automorphisms of $F$. (Hint: start by computing the ring of endomorphisms of $F$ as a functor without regard to the tensor structure).

(b)$^*$ Let $H \subset G$ be a subgroup. Compute the group $\mathrm{Aut}^{\otimes}(\mathrm{Res}_H^G)$ of the restriction functor.

4. Give an example of non-isomorphic tensor functors which are isomorphic as functors. (Hint: think about pointed categories).

5. Let $\mathcal{C}$ be a monoidal category and let $X, Y \in \mathcal{C}$ be two objects such that $X \otimes Y \simeq Y \otimes X \simeq \mathbf{1}$. Prove that $X$ is invertible.