**TOPOLOGICAL FIELD THEORIES AND TENSOR CATEGORIES. HOMEWORK #4.**

## Target day for this homework: November 25

Submit (in class or by email) any of the following problems.

1. Let $A$ be a finite dimensional algebra over a field of characteristic zero. Prove that $A$ is semisimple if and only if the scalar product $\mathrm{tr}\,(xy)$ is non-degenerate (here $\mathrm{tr}\,(x)$ is the trace of linear transformation $a \mapsto xa$ of $A$ to itself). Hint: think about Jacobson radical.

2. (a) Prove that an exact tensor functor between rigid abelian tensor categories with indecomposable unit objects is faithful. Hint: think about the image of a morphism annihilated by the functor.

(b) Give an example of tensor functor between rigid abelian tensor categories with indecomposable unit objects which is not faithful (and hence non-exact).

3. (a) Show that the category $\mathrm{Rep}(O_t), t \notin \mathbb{Z}$ over a field of characteristic zero is not of moderate growth.

(b) Show that a rigid abelian tensor category with finitely many simple objects and finite length of any object is of moderate growth. Hint: think about matrix of multiplication by an object in the Grothendieck ring.

4. Find a mistake in the statement of Proposition 9.8 of Deligne's paper about the symmetric group $S_t$. What is a correct statement? You might need to look at the paper by H.Wenzl about Brauer algebras.

5. (a) This is computer programming exercise: Compute the determinant of the trace form on the Brauer algebra for $m = 3$ (and $m = 4$ if you are really brave).

(b) What are dimensions of walled Brauer algebras? Compute the determinants of the trace form in some low-dimensional cases.

(c) Modify your program from (a) to to computations with walled Brauer algebras and make some computations.