TENSOR CATEGORIES. HOMEWORK #1.

Target day for this homework: February 13

Submit (in class or by email) any of the following problems.

1. Make precise and prove the following statement: linear functors and multi-functors from a semisimple category are determined (up to natural isomorphism of functors) by their values.

2. Let $(\mathcal{C}, \otimes, a)$ be a semigroup category (i.e. category with a tensor product and an associativity constraint satisfying a pentagon axiom). Let $S \in \mathcal{C}$ be an object such that $S \otimes S \sim S$ and the functors $X \mapsto S \otimes X$ and $X \mapsto X \otimes S$ are fully faithful. Prove that S is the unit object of $\mathcal{C}$. (cf Lemma 4.2.6 of [KS]).

3. Recall pointed tensor categories Vec_G^ω. Prove that there exists a surjective tensor functor $\mathrm{Vec}_{C_4} \to \mathrm{Vec}_{C_2}^\omega$ where ω is nontrivial.

4. Assume that (F, J) is a monoidal functor $\mathcal{C} \to \mathcal{D}$ such that F is an equivalence. Prove that there exists a *monoidal* functor $(G, J') : \mathcal{D} \to \mathcal{C}$ such that both compositions $F \circ G$ and $G \circ F$ are isomorphic to the identity functor(s) as monoidal functors.

5. Give a description of pointed categories in terms of *normalized* cocycles (see e.g. Exercise 2.3.9 in [EGNO]).