TENSOR CATEGORIES. HOMEWORK #2.

Target day for this homework: March 5

Submit (in class or by email) any of the following problems.

1. Let $\mathcal{C}$ and $\mathcal{D}$ be two *rigid* monoidal categories and let $F, G : \mathcal{C} \to \mathcal{D}$ be two monoidal functors. Prove that any morphism of monoidal functors $F \to G$ is an isomorphism (see Exercise 2.10.15 in [EGNO] or https://ncatlab.org/toddtrimble/published/Morphisms+between+tensor+functors).

2. Let V be the irreducible 2-dimensional complex representation of the dihedral group D_8 of order 8. Compute the number $\lambda(V)$ associated with this representation. Repeat this problem when D_8 is replaced by the quaternion group Q_8. Deduce that the monoidal categories $\mathrm{Rep}_{\mathbb{C}}(D_8)$ and $\mathrm{Rep}_{\mathbb{C}}(Q_8)$ are not equivalent.

3. Let $\mathcal{C}$ be a monoidal category (perhaps non-additive).

(a) Define most obvious monoidal structure on the category $\mathcal{C} \times \mathcal{C}$. Show that $\mathcal{C}$ is braided if and only if the tensor product functor $\mathcal{C} \times \mathcal{C} \to \mathcal{C}$ is a monoidal functor (this means that braidings are in bijection with the structures of monoidal functor).

(b) Assume $\mathcal{C}$ is braided. Define most obvious braided structure on the category $\mathcal{C} \times \mathcal{C}$. Show that is $\mathcal{C}$ symmetric if and only if the tensor product functor $\mathcal{C} \times \mathcal{C} \to \mathcal{C}$ is braided.

4. Let A be a finite abelian group and let $q : A \to \mathbb{C}^\times$ be a quadratic form. Let $\mathcal{C}(A, q)$ be the corresponding pointed braided tensor category over $\mathbb{C}$ (which exists by Eilenberg-Maclane theorem). Prove that the associator on the category $\mathcal{C}(A, q)$ is trivial if and only if there exists a bicharacter $B : A \times A \to \mathbb{C}^\times$ such that $q(x) = B(x, x)$ for all $x \in A$ (see Exercise 8.4.11 in [EGNO]).

5. Prove that the two hexagon axioms in the definition of a braided monoidal category are independent, that is none of them imply the other one. Hint: think about pointed categories with trivial associator.