Schedule of workshop on Topics of Enumerative Geometry

- Conference room: Fenton 110.
- Coffee/Reception: Fenton 219.
- Coffee/pastries 8:45AM & 2:30PM. Reception: 6PM-8PM. All are welcome!
- Sunday afternoon: a hike from Martin Street trailhead to Spence butte.

Schedule, Title/abstract

Saturday Morning, May 21

9:15-10:15 Chiu-Chu Melissa Liu, Higgs-Coulomb correspondence for abelian gauged linear sigma models

Abstract: The input data of a gauged linear sigma model (GLSM) consists of a GIT quotient of a complex vector space V by the linear action of a reductive algebraic group G (the gauge group) and a G-invariant polynomial function on V (the superpotential) which is quasi-homogeneous with respect to a \mathbb{C}^* -action (R symmetries) on V. The Higgs-Coulomb correspondence relates (1) GLSM invariants which are virtual counts of Landau-Ginzburg quasimaps (Higgs branch), and (2) Mellin-Barnes type integrals on the Lie algebra of G (Coulomb branch). In this talk, I will describe the correspondence when G is an algebraic torus, and explain how to use the correspondence to study dependence of GLSM invariants on the stability condition. This is based on joint work in progress with Konstantin Aleshkin.

10:45-11:45 Tony Yue Yu, Non-archimedean Quantum K-theory and Gromov-Witten invariants

Abstract: Motivated by mirror symmetry and the enumeration of curves with boundaries, it is desirable to develop a theory of Gromov-Witten invariants in the setting of non-archimedean geometry. I will explain our recent works in this direction. Our approach differs from the classical one in algebraic geometry via perfect obstruction theory. Instead, we build on our previous works on the foundation of derived non-archimedean geometry, the representability theorem and Gromov compactness. We obtain numerical invariants by passing to K-theory or motivic cohomology. We prove a list of natural geometric relations between the stacks of stable maps, directly at the derived level, with respect to elementary operations on graphs, namely, products, cutting edges, forgetting tails and contracting edges. They imply the corresponding properties of numerical invariants. The derived approach produces highly intuitive statements and functorial proofs. Furthermore, its flexibility allows us to impose not only simple incidence conditions for marked points, but also incidence conditions with multiplicities. Joint work with M Porta.

Saturday Afternoon, May 21

1:30-2:30 Dragos Oprea, On the tautological vector bundles over Hilbert and Quot schemes

Abstract: The Hilbert scheme of points on surfaces and the punctual Quot schemes on curves carry tautological vector bundles. I will report on: (i) the calculation of holomorphic Euler characteristics of tautological vector bundles over the punctual Quot schemes of curves (based on joint work with Shubham Sinha), and (ii) (time permitting) results concerning the positivity properties of tautological bundles over Hilbert and Quot schemes.

3:00-4:00 Junliang Shen, From curve counting on K3 surfaces to holomorphic symplectic geometry

Abstract: Considerations in physics provide us a list of interesting formulas (Yau-Zaslow, Katz-Klemm-Vafa etc) concerning counting invariants of curves on a K3 surface. Rich geometries have been developed to "explain" these equations. In this talk, we will focus on the interplay between the enumerative geometry of K3 and holomorphic symplectic geometry (in particular topology of Lagrangian fibrations). We discuss this connection from both the perspectives of cohomology and derived categories. This is based on joint work with Qizheng Yin. If time permits, I will also discuss joint work in progress with Davesh Maulik and Qizheng Yin on a generalization of the picture above involving Fourier-Mukai transforms and the decomposition theorem for Lagrangian fibrations.

4:30-5:30 Mark Shoemaker, Towards a mirror theorem for GLSMs

Abstract: A gauged linear sigma model (GLSM) consists roughly of a complex vector space V, a group G acting on V, a character θ of G, and a G-invariant polynomial w on V. This data defines a GIT quotient Y = [V//G] and a regular function on that quotient. GLSMs arise naturally in a number of contexts, for instance as the mirrors to Fano manifolds and as examples of noncommutative crepant resolutions. GLSMs provide a broad setting in which it is possible to define an enumerative curve counting theory, simultaneously generalizing FJRW theory and the Gromov-Witten theory of hypersurfaces. Despite a significant effort to rigorously define the enumerative invariants of a GLSM, very few computations of these invariants have been carried out. In this talk I will describe a new method for computing generating functions of genus zero GLSM invariants. I will explain how these generating functions arise as derivatives of generating functions of Y.

Sunday Morning, May 22

9:00-10:00 Andrei Căldăraru, Categorical Enumerative Invariants and the λ_g conjecture

Abstract: I will give an overview of the theory of Categorical Enumerative Invariants (CEIs), as developed in joint work with Junwu Tu and Kevin Costello. The emphasis will be on the role played by the Givental group, which controls the effect of changes of splitting on the CEIs. If time allows I will state two new conjectures which relate the derived category of the nodal cubic with the classical λ_g conjecture in Gromov-Witten theory, due to Getzler, Faber and Pandharipande. The new work is joint with Weng-Him Cheung and Yunfan He, and builds on work of Yefeng Shen and Junwu Tu (unpublished).

10:20-11:20 Guangbo Xu, Integer-valued Gromov-Witten type invariants

Abstract: Gromov-Witten invariants for a general target are rational-valued but not necessarily integervalued. This is due to the contributions of curves with nontrivial automorphism groups. In 1997 Fukaya and Ono proposed a new method in symplectic geometry which can define integer counts of curves with a trivial automorphism group. While ordinary Gromov-Witten invariants only use the orientation on the moduli spaces, these integer-valued counts are supposed to also use the (stable) complex structure on the moduli spaces. In this talk I will present the recent joint work with Shaoyun Bai in which we rigorously defined the integer-valued Gromov-Witten type invariants in genus zero for a symplectic manifold following Fukaya-Ono's proposal. I hope algebraic geometers can provide ideas on how to construct these invariants algebraically, how to describe the mirror symmetry about these counts, or other interesting aspects.

11:40-12:40 François Greer, Cycle-valued quasi-modular forms

Abstract: Arithmetic quotients of Type IV Hermitian symmetric domains have cohomology-valued modular forms whose coefficients are special cycles by work of Borcherds. These can be interpreted as non-compact period spaces for K3-type Hodge structures. I will describe recent results (joint with P. Engel and S. Tayou) that give mock modular forms whose coefficients are compactified special cycles in a simplicial toroidal compactification. Next, I will give two conjectural analogues of this result: one on the Gromov-Witten theory of elliptic fibrations, and one on the moduli space of principally polarized abelian varieties.