1. Section 34

34.2 Calculate
(a) \( \lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2} \, dt \)  
(b) \( \lim_{h \to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} \, dt \).

34.3 Let \( f \) be defined as follows: \( f(t) = 0 \) for \( t < 0 \); \( f(t) = t \) for \( 0 \leq t \leq 1 \); \( f(t) = 4 \) for \( t > 1 \).
(a) Determine the function \( F(x) = \int_0^x f(t) \, dt \).
(b) Sketch \( F \). Where is \( F \) continuous?
(c) Where is \( F \) differentiable? Calculate \( F' \) at the points of differentiability.

34.6 Let \( f \) be a continuous function on \( \mathbb{R} \) and define
\[
G(x) = \int_0^{\sin x} f(t) \, dt \quad \text{for} \quad x \in \mathbb{R}.
\]
Show \( G \) is differentiable on \( \mathbb{R} \) and compute \( G' \).

34.11 Suppose \( f \) is a continuous function on \([a, b]\). Show that if \( \int_a^b f(x)^2 \, dx = 0 \), then \( f(x) = 0 \) for all \( x \) in \([a, b]\). \textit{Hint:} See Theorem 33.4.

34.12 Show that if \( f \) is a continuous real-valued function on \([a, b]\) satisfying \( \int_a^b f(x)g(x) \, dx = 0 \) for every continuous function \( g \) on \([a, b]\), then \( f(x) = 0 \) for all \( x \) in \([a, b]\).