Besse 2018, Lecture III

Main Exercise 1. Compute the double recursion for $\mathfrak{gl}_3$ clasps, and verify that the conjecture solves the recursion in this case. Where does the dominance order on weights seem to play a role in the recursive formulas?

Main Exercise 2. Compute the cellular form for $\underline{i} = (1,1,1,1,1)$ at the weights $5\omega_1$, $3\omega_1 + \omega_2$, and $\omega_1 + 2\omega_2$. (Hint: Your form should be $\pm$-definite, which is a shadow of Hodge theory!)

Besse 2018, Lecture III supplementary exercises

Exercise 1. Compute the double recursion for $\mathfrak{gl}_4$ clasps, and verify that the conjecture solves the recursion in this case.

Exercise 2. Fix a field $\Bbbk$. Let B be an absolutely indecomposable object in a $\Bbbk$-linear Krull-Schmidt category $\mathcal{C}$; this means that its endomorphism ring is local, with maximal ideal $\mathfrak{m}$, and $\mathrm{End}(B)/\mathfrak{m}$ is spanned by the identity map. Let X be any other object in $\mathcal{C}$. Then one has a pairing

$$\mathrm{Hom}(X, B) \times \mathrm{Hom}(B, X) \to \mathrm{End}(B)/\mathfrak{m} = \Bbbk$$

which we call the *local intersection pairing* of X at B. Prove that the multiplicity of B as a direct summand of X is equal to the rank of the local intersection pairing. Why does the local intersection pairing match up with the cellular pairing for an OACC?