See the other document for a list of topics.

This is not a true practice final. The previous midterms and practice midterms have many representative problems covering their topics. I have only tried to give representative problems which have not been covered by previous exams. I strongly recommend looking over the previous material as well.

I have not attempted to make this practice final an appropriate length. You should expect there to be 12 questions on the actual final, with overall length a little more than twice the usual midterm. I'm expecting two problems to be on ODEs and one problem to be partly on recognizing common Taylor series when evaluated. The ODE problems are worded differently than in the homework and quizzes in order to make partial credit easier to obtain.

1. Evaluate the following series exactly if they converge, or explain why they diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{3 \cdot 2^{3n+2}}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{5 \cdot 2^n}{7}$

   (c) $\displaystyle\sum_{n=4}^{\infty} \frac{1}{\ln(n^2)} - \frac{1}{\ln((n+1)^2)}$

   (d) $\displaystyle\sum_{n=4}^{\infty} \ln(n^2) - \ln((n+1)^2)$

   (e) $4 + 12 + \dfrac{36}{2} + \dfrac{108}{6} + \dfrac{4 \cdot 81}{24} + \dots$

   (f) $7 - \dfrac{7^2}{2} + \dfrac{7^3}{3} - \dfrac{7^4}{4} + \dots$

(g) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{(13)^{2n}}{(2n)!}+(-1)^n\frac{9^{2n+1}}{(2n+1)!}$

2. Let $y=\sum_{n=0}^{\infty}a_nt^n$ be a power series centered at 0.

   (a) Write down power series centered at 0 for y', y'', and y'''.

   (b) Suppose that $y(0)=2$, $y'(0)=1$, and $y''(0)=6$. What does that say about the coefficients a_n?

   (c) Suppose that y solves the differential equation $y'''-2y''+y'-2y=0$ with the initial conditions given above. Find a_n for $n\leq 4$.

3. Let $y = \sum_{n=0}^{\infty} a_n(t-6)^n$ be a power series centered at 6.

   (a) Write down power series centered at 6 for y' and $(t+2)y$.

   (b) Suppose that y solves the differential equation $y' - (t+2)y = 15$. Write down a recursive formula for the coefficients a_n.

   (c) Find the first three non-zero terms of the general solution centered at 6.

4. Suppose that y solves the differential equation $y'' + ty' + y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 0$.

   (a) Suppose that $y = \sum a_n t^n$. Find a recursive formula for the coefficients a_n.

   (b) Find the values of a_n for $n \leq 8$.

   (c) Approximate the value of $y(0.1)$ to within 10^{-6}. Justify your answer.