Approximate List of Topics:

- **Sequences** Convergence and divergence - what they mean. The mathematical definition of a limit. What it means to go to infinity.
- Testing for convergence: squeeze theorem, monotone sequence theorem, geometric sequences
- Different ways to write down sequences (explicit, recursive, etc)
- Finding limits: plugging in continuous functions, L'Hopital's rule. Limits of recursive sequences.
- **Series** Convergence and divergence - what they mean. The difference between a sequence and a series.
- Geometric series and p-series.
- Testing for convergence: telescoping sums, the "test for divergence".
- Series with positive entries: the integral test, the comparison test, the limit comparison test.
- Series with positive and negative entries: definition of absolute convergence, the absolute convergence test, the alternating series test.
- Remainder estimates: for integral test, and for alternating series test.

This is too long for an actual midterm... but not much too long. If you can't do this in a bit over an hour, you need more practice.

1. Does the sequence converge or diverge? WHY? If it converges, what is the limit.

   (a)

$$a_n = \sqrt{\frac{4n^2 - 2n + 3}{2n^2 + 17}}$$

   (b)

$$b_n = \frac{n^n}{n!}$$

   (c)

$$c_n = (\frac{1}{2}, \frac{-1}{2}, \frac{1}{3}, \frac{-2}{3}, \frac{1}{4}, \frac{-3}{4}, \frac{1}{5}, \frac{-4}{5}, \ldots)$$

   (d)

$$d_n = 8000(\frac{99}{100})^n$$

   (e)

$$e_n = \frac{\ln n}{\ln(\ln n)}$$

2. Consider the sequence $a_n = \frac{n^2-2}{n^2}$.
   (a) How many terms of the sequence are needed before it gets within $.01$ of the limit? Within $.001$ of the limit?
   (b) Using the mathematical definition of limit, prove that the sequence converges.

3. Consider the sequence $a_1 = 10$ and $a_{n+1} = \frac{1}{4}a_n + 3$.
   (a) Show that the sequence is monotone and bounded, and therefore converges.
   (b) What is the limit?

4. Does the series converge or diverge? WHY?

(a)

$$\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.05}} + \frac{1}{n^{.95}}$$

(c)

$$\sum_{n=1}^{\infty} \frac{n10^n}{3^{3n+2}}$$

(d)

$$\sum_{n=1}^{\infty} \sin(n) - \sin(n+1)$$

(e)

$$\sum_{n=0}^{\infty} \frac{n^2+2}{n^3+3}$$

(f)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

(g)

$$-1-\frac{1}{4}+\frac{1}{9}+\frac{1}{16}-\frac{1}{25}-\frac{1}{36}+\frac{1}{49}+\dots$$

5. Consider the series

$$\frac{1}{4}-\frac{1}{8}+\frac{1}{12}-\frac{1}{16}+\dots$$

(a) Is the series convergent? WHY?

(b) How many terms of the sum must one take in order to be within .1 of the limit?

(c) Is the series absolutely convergent? WHY?

6. What is the sum of the following series?

$$\sum_{n=6}^{\infty}\left(\frac{6}{7}\right)^n$$

7. How many terms of the series $\sum_{n=1}^{\infty} \frac{4}{n^5}$ must you take to approximate the sum of the series to within 10^{-8}?