Approximate List of Topics:

- **Series** The ratio test.
- **Power Series** Finding the radius of convergence (using the ratio test). Finding the interval of convergence (using other tests on the boundary).
- Manipulation of power series: addition, multiplication, derivatives, integrals. Using the geometric series.
- Using power series and series bounding methods to estimate values and integrals.
- **Taylor Series** Understanding what it means to approximate a function around a point c to degree k.
- The Taylor series at 0 for common functions: $\frac{1}{1-x}$, $\ln(x+1)$, $\sin x$, $\cos x$, e^x, $\arctan(x)$, $(1+x)^k$.
- Computing derivatives to compute the Taylor series around any point, or to compute the k-th order approximation.
- Taylor's inequality estimate!!!!!!!!!!!!!
- Proving that a Taylor series converges to a function.

This is definitely too long for an actual midterm!! I decided that more problems is more useful. If you want to time yourself, imagine that I will assign only 6 problems.

1. What does the **ratio test** say about the following series?

   (a) $\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n!}$

   (b) $\sum_{n=1}^{\infty}\frac{n^2+3}{n^3+2}$

2. Find the interval of convergence of the following power series.

   (a) $\sum_{n=0}^{\infty}\frac{(x-2)^n}{n3^n}$

   (b) $\sum_{n=0}^{\infty}\frac{4^n(x+9)^n}{n^3+1}$

3. Find a power series centered at zero for the following functions. (Note: I could also ask for the radius of convergence.)

   (a) $\dfrac{1}{4-3x}$

   (b) $\displaystyle\int_0^x \frac{1}{1+t^3}\,dt$

   (c) The derivative of $\displaystyle\sum_{n=0}^{\infty} \frac{2^n (n!) x^n}{(3n)!}$.

4. Compute $\int_0^{1/10} \frac{1}{1+t^3}\,dt$ to within 10^{-9}.

5. Find a power series centered at zero for the following functions. Write out the first three nonzero terms explicitly. (Note: I could also ask for the radius of convergence.)

   (a) e^{x^3}

   (b) $\dfrac{1}{(1+2x)^{3.5}}$

   (c) $\ln(1-x^3)$

6. Find $\cos(.5)$ to within $\frac{1}{500}$. Use the Taylor Inequality Estimate to justify your answer.

7. Using any method, find the first few terms of the Taylor series, up to the cubic term (i.e. the x^3 term).

   (a) $e^x \cos x$ centered at 0.

   (b) $\sqrt{x-3}$ centered at 2.

   (c) e^{3x} centered at -5.

8. Find the third-order approximation to $\frac{1}{1-x}$ at 5. Bound the error on the interval $(4, 6)$.

9. Find the fifth-order approximation to $3\sin x$ at 0. Use the Taylor Inequality Estimate to find the radius d such that the error is less than $.2$ for x in the interval $(-d, d)$.

10. Prove that the Taylor series of e^x centered at 2 will converge to the function e^x everywhere.