Answer all questions in the space provided.
Approximate List of Topics:

- Basics + Definitions. What is an ODE? solution? IVP? order? system of ODEs?
- GIVEN A 1ODE, WHAT CAN YOU DO WITH IT?
- Existence and uniqueness theorem for 1ODEs. Domain of definition; what is special about linear ODEs.
- Solving some 1ODEs: separable, linear. Implicit vs. explicit solutions.
- Linear operators. Homogeneous vs. inhomogeneous.
- Understanding autonomous ODEs. Equilibria and stability.
- Euler method for numerical approximation.
- Direction fields and qualitative analysis. Isoclines, funnels, separatrices. Finding maxima and minima.

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General notes:

This is way longer than the actual midterm. The actual midterm will have 4 questions, and each one will probably have fewer parts. Time yourself, but aim for something more like 70-80 minutes or so.

The question "What kind of differential equation is this?" will occur often. If it is an ODE, always mention the order. If it is a special kind of ODE, say so. If it is linear, say whether it is homogeneous or not.

Examples: "This is a separable 1ODE" or "This is an inhomogeneous 3LODEwCC" or "This is a (plain) 5ODE" or "This is a PDE" or "This is an autonomous 1ODE."

The question "Find the general solution." will occur often. You will get partial credit for knowing what to do. Solutions should be left in implicit form when finding an explicit form is difficult.

Sometimes, it is a trick question! If it is not an ODE that you are supposed to know how to solve, say so for full credit.

1. Consider the differential equation $y' = 3t^2y + e^{t+t^3}$.

   (a) What kind of differential equation is this?

   (b) Find the general solution.

   (c) Verify that your general solution satisfies the differential equation. (Show your work).

   (d) Find a solution for which $y(2) = 0$.

   (e) Is the solution you found in part (d) unique?

2. Consider the differential equation $y' = -.001(y+3)(y-2)^2(y-4)^3$.

   (a) What kind of differential equation is this?

   (b) Find all the equilibrium solutions, and classify their stability.

   (c) On one graph, sketch multiple solutions to this differential equation. Sketch all the equilibrium solutions, and at least two solutions with each possible pattern of behavior. (Do not worry about getting the slopes precise, just the general behavior! Aside: This is the difference between the word "sketch" and the word "draw.")

   (d) Suppose that $y(0) = a$ and we plan to use Euler's method with step size 1 to estimate $y(4)$. For what values of a, approximately, should one worry that our estimate is extremely bad? Explain.

   (e) Start solving this differential equation. You may stop when you have reached a difficult integral to compute. Say what method you would use to compute this integral. (But do not compute it!)

3. Consider the differential equation $y' = 2y(t + y)$.

   (a) What kind of differential equation is this?

   (b) Find the general solution.

   (c) Suppose that $y(0) = -0.5$. Use Euler's method with step size 1 to estimate $y(3)$.

   (d) Suppose that $y(2) = 1$. Use Euler's method with step size 0.5 to estimate $y(3)$.

4. Consider the differential equation $y' = 2(y-1)^2 e^{2t}$.
   (a) What kind of differential equation is this?
   (b) Find the general solution.
   (c) Suppose that $y(0) = 2$. Where is this solution defined?
   (d) Suppose that $y(0) = \frac{1}{2}$. Where is this solution defined?
   (e) (EXTRA CREDIT): For which a will the solution satisfying $y(0) = a$ be defined everywhere? Which solutions are separatrices?

5. Consider the differential equation $y' = y^2 - x$.

   (a) What kind of differential equation is this?

   (b) Draw the direction field of this differential equation, in the range from $-3 \leq y \leq 3$ and $-1 \leq x \leq 4$.

   (c) On your graph, draw the isoclines with slope -1, 0, and $+1$.

   (d) On your graph, draw the solutions through $y(1) = a$ for $a = -2, -1, 0, 1, 2$. How many local maxima does each solution have? How many local minima?

   (e) For the solution with $y(1) = -2$, estimate $y(100)$ to within $\pm.5$. Explain your estimate using funnels.

   (f) On your graph, draw the separatrix. Estimate $y(100)$ for the separatrix.

6. Bonus problems (the test is long enough, these are for more practice). For each of these: what kind of differential equation is it? Find the general solution. For $y(0) = a$, can you figure out where the solution is defined (if so, do it). What techniques can you apply (e.g. solving, approximation, etc)?

   (a) Consider the differential equation $y' = \frac{3x^2+2x}{4y^3+2y}$.

   (b) Consider the differential equation $y' = t^y y^t \sin(\ln(t+y))$.

   (c) Consider the differential equation $y' = e^{t^2} y + e^{\sin t}$.

   (d) Consider the differential equation $y'' = yy' + t$.

   (e) Consider the differential equation $y' = \frac{y^2}{y^2+1} + 10y + t^2$, assuming that $y(0) > 10$.

   (f) Consider the differential equation $\frac{d^2z}{dt^2} = e^t \frac{dz}{dx} + z(x,t)\frac{dz}{dt}$.