

Answer all questions in the space provided.

**Approximate List of Topics:**

- GIVEN A nODE, WHAT CAN YOU DO WITH IT?
- Existence and uniqueness theorem for nODEs. What is an IVP? Domain of definition; what is special about linear ODEs.
- Properties of linear operators. Superposition. Homogeneous and inhomogeneous solution spaces.
- Solving homogeneous nLODEs with constant coefficients.
- Solving inhomogeneous nLODEs with constant coefficients, and nice forcing functions.
- Solving initial value problems for the above.
- Exponential response formula.
- Complex numbers. Going between rectangular and polar coordinates. Euler's formula.
- Rewriting sinusoidal functions. Amplitude and phase lag.

**General notes:**

The question "What kind of differential equation is this?" will occur often. If it is an ODE, always mention the order. If it is a special kind of ODE, say so. Examples: "This is a separable 1ODE" or "This is a 3ODE" or "This is a PDE" or "This is an autonomous 1ODE" or "This is an inhomogeneous 5LODEwCC."

The question "Find the general solution." will occur often. Sometimes, it is a trick question!!!!!! If it is not an ODE that you are supposed to know how to solve, and you say so, then you get full credit! If you are supposed to know how, and you begin or outline the process, you will get some partial credit at least.

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1. (a) Rewrite  $-\sqrt{3} + i$  in the form  $Ae^{i\theta}$ .

(b) Rewrite  $5e^{i \arctan(\frac{3}{4})}$  in the form  $a + bi$ .

(c) Rewrite  $-\cos(3t) - 3\sin(3t)$  in the form  $C \cos(\omega t - \varphi)$ . What is the amplitude of this sinusoidal function?

(d) Compute  $\frac{2-3i}{3-4i}$  in either rectangular or polar form.

(e) Compute  $\frac{\sqrt{5}e^{\frac{3}{4}\pi i}}{4e^{-\frac{5}{8}\pi i}}$  in either rectangular or polar form.

2. Consider the differential equation  $y'' - 4y' + 29y = 5e^t - 3e^{-2t}$ .

(a) What kind of differential equation is this?

(b) Find the general solution.

(c) Find a solution for which  $y(0) = 1$  and  $y'(0) = 2$ .

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3. Let  $\mathcal{L}$  denote the operator (on functions in a variable  $t$ ) defined by

$$\mathcal{L}[y] = y'' + (1000 - \sin(3t))y' - 2y.$$

- (a) Is  $\mathcal{L}$  a linear operator?
- (b) What kind of differential equation is  $\mathcal{L}[y] = 0$ ?
- (c) Let  $f = \mathcal{L}[e^{2t}]$ . Find  $f$ .
- (d) Find a solution to  $\mathcal{L}[y] = f$ .
- (e) Find the general solution to  $\mathcal{L}[y] = f$ .
- (f) Suppose that  $z$  is a solution to  $\mathcal{L}[y] = f$  satisfying  $z(5) = 6$  and  $z'(5) = -3$ . Does  $z$  exist? Where is  $z$  defined? Why?
- (g) Find a linear operator  $\mathcal{L}_2$  for which the solutions to  $\mathcal{L}_2[y] = f$  approximate the solutions to  $\mathcal{L}[y] = f$ , and for which you know how to solve  $\mathcal{L}_2[y] = f$ . You don't need to find the solutions.

4. Let  $\mathcal{L} = p(D)$  for the polynomial  $p(x) = x^2(x - 1)(x + 2)^3(x^2 - 2x + 2)^2(x^2 + 5x + 1)$ . Here,  $D$  represents the derivative operator.

(a) What kind of differential equation is  $\mathcal{L}[y] = e^{5t}$ ?

(b) Find the general homogeneous solution.

(c) Write down a guess for a particular solution to  $\mathcal{L}[y] = (3t - 5)e^{5t}$ . You need not determine any coefficients.

(d) Write down a guess for a particular solution to  $\mathcal{L}[y] = 16e^{-2t}$ . You need not determine any coefficients.

(e) Write down a guess for a particular solution to  $\mathcal{L}[y] = \cos t$ . You need not determine any coefficients.