

Answer all questions in the space provided.

Approximate List of Topics:

- Basics + Definitions. What is an ODE? solution? IVP? order? system of ODEs?
- GIVEN A 1ODE, WHAT CAN YOU DO WITH IT?
- Existence and uniqueness theorem for 1ODEs. Domain of definition; what is special about linear ODEs.
- Solving some 1ODEs: separable, linear. Implicit vs. explicit solutions.
- Understanding autonomous ODEs. Equilibria and stability.
- Euler method for numerical approximation.
- Direction fields and qualitative analysis. Isoclines, funnels, separatrices. Finding maxima and minima.
- Linearizing an ODE.

General notes:

The question "What kind of differential equation is this?" will occur often. If it is an ODE, always mention the order. If it is a special kind of ODE, say so. Examples: "This is a separable 1ODE" or "This is a 3ODE" or "This is a PDE" or "This is an autonomous 1ODE."

The question "Find the general solution." will occur often. Sometimes, it is a trick question!!!!!! If it is not an ODE that you are supposed to know how to solve, and you say so, then you get full credit! If you are supposed to know how, and you begin or outline the process, you will get some partial credit at least.

1. Consider the differential equation $y' = 3t^2y + e^{t+t^3}$.

(a) What kind of differential equation is this?

(b) Find the general solution.

(c) Verify that your general solution satisfies the differential equation. (Show your work).

(d) Find a solution for which $y(2) = 0$.

(e) Is the solution you found in part (d) unique?

3. Consider the differential equation $y' = 2y(t + y)$.

(a) What kind of differential equation is this?

(b) Find the general solution.

(c) Suppose that $y(0) = -0.5$. Use Euler's method with step size 1 to estimate $y(3)$.

(d) Suppose that $y(2) = 1$. Use Euler's method with step size 0.5 to estimate $y(3)$.

4. Consider the differential equation $y' = 2(y - 1)^2 e^{2t}$.

(a) What kind of differential equation is this?

(b) Find the general solution.

(c) Suppose that $y(0) = 2$. Where is this solution defined?

(d) Suppose that $y(0) = \frac{1}{2}$. Where is this solution defined?

(e) (EXTRA CREDIT): For which a will the solution satisfying $y(0) = a$ be defined everywhere? Which solutions are separatrices?

5. Consider the differential equation $y' = y^2 - x$.

- (a) What kind of differential equation is this?
- (b) Draw the direction field of this differential equation, in the range from $-3 \leq y \leq 3$ and $-1 \leq x \leq 4$.

- (c) On your graph, draw the isoclines with slope -1 , 0 , and $+1$.
- (d) On your graph, draw the solutions through $y(1) = a$ for $a = -2, -1, 0, 1, 2$. How many local maxima does each solution have? How many local minima?
- (e) For the solution with $y(1) = -2$, estimate $y(100)$ to within ± 0.5 . Explain your estimate using funnels.
- (f) On your graph, draw the separatrix. Estimate $y(100)$ for the separatrix.

6. Bonus problems (the test is long enough, these are for more practice). For each of these: what kind of differential equation is it? Find the general solution. For $y(0) = a$, can you figure out where the solution is defined (if so, do it). What techniques can you apply (e.g. solving, approximation, etc)?

(a) Consider the differential equation $y' = \frac{3x^2+2x}{4y^3+2y}$.

(b) Consider the differential equation $y' = t^y y^t \sin(\ln(t + y))$.

(c) Consider the differential equation $y' = e^{t^2} y + e^{\sin t}$.

(d) Consider the differential equation $y'' = yy' + t$.

(e) Consider the differential equation $y' = \frac{y^2}{y^2+1} + 10y + t^2$, assuming that $y(0) > 10$.

(f) Consider the differential equation $\frac{d^2 z}{dt^2} = e^t \frac{dz}{dx} + z(x, t) \frac{dz}{dt}$.