Math 341 (Elementary Linear Algebra I), Winter 2014
Practice Midterm 1

October 14, 2014

Answer all questions in the space provided.

1. For each of the following statements, indicate whether the statement is true or false. If it is false, briefly explain why or give a counter-example.

   (a) If a coefficient matrix has a column of zeroes, then the corresponding linear system has no solutions.

   (b) A linear system with more variables than equations has either 0 or infinitely many solutions, never a unique solution.

   (c) It is possible for a linear system two have exactly three solutions.

   (d) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are vectors in $\mathbb{R}^2$, then it is always possible to write $\mathbf{v}_3$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

   (e) Swapping two columns (other than the last column) of an augmented matrix does not change the solution set of the corresponding linear system.

   (f) If the fourth column of a matrix is a pivot column, then so is the third column.

2. You run a company which manufactures two products: Gadgets and Widgets.
   Every Gadget costs your company 4 dollars in parts, 2 dollars in labor, and 1 dollar of electricity. Every Widget costs your company 2 dollars in parts, 8 dollars in labor, and 3 dollars of electricity.
   At the end of the year, your accountant's records say that you have spent 15 million dollars in parts, 6 million dollars in labor, and 2 million dollars of electricity.

   (a) Provide a linear system which governs the number of Gadgets and Widgets produced by your company. Label your equations and your variables. You do not need to find the solutions, just provide the setup.

   (b) Write this linear system in matrix equation form.

   (c) Should you fire your accountant? Why or why not?

3. Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be vectors in $\mathbb{R}^n$. State the definition of "linear combination" of these vectors. State the definition of $\text{span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$.

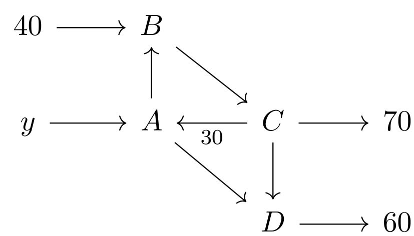
4. Solve for all solutions of the linear system below using row reduction. At each step you must say which rule(s) you are using. When done, state the number of solutions. *Do not simply use substitution!*

$$\begin{aligned} 3x_1 \qquad + x_3 &= -1 \\ - x_2 - x_3 &= 1 \\ x_1 + 4x_2 + 2x_3 &= -9 \end{aligned}$$

5. Solve for all solutions of the linear system below. When done, state the number of solutions. *Do not simply use substitution!*

$$\begin{aligned} x_1 \qquad + 7x_3 \qquad + x_5 &= 3 \\ x_2 + 2x_3 \qquad - x_5 &= 3 \\ x_4 + 6x_5 &= 2 \end{aligned}$$

6. Consider the network flow problem below.



(a) Set up a linear system which describes the flow. (You may choose whether to consider y as a variable, or as part of the target vector.)

(b) For which values of y is the system consistent?

(c) In a consistent flow, what is the minimum value for the edge from point A to point B?