Math 431/531 (Topology), Fall 2015
HW 3

Starred problems are for 531 students, and are extra credit for 431 students. 531 students must LaTeX their solutions.

Many of these are examples (e.g. find the closure of this, sort these things), for which I just want the answer, not a proof that you're correct.

1. Exercises 4.2 (just the alphabet), 4.5(cd*e) from K.

2. 2, 3 from Munkres, p92.

3. 5*, 6, 7, 20(abcd) from Munkres, p101-2.

4. (Holdout from last time) Consider the following topologies on $\mathbb{R}$:

   (a) The standard topology
   (b) The discrete topology
   (c) The indiscrete topology
   (d) The topology $\mathbb{R}_K$ for $K = \{\frac{1}{n}\}_{n\geq 2}$
   (e) The topology $\mathbb{R}_{-\infty}$
   (f) The topology $\mathbb{R}_l$
   (g) The topology $\mathbb{R}_{\mathbf{0}}$
   (h) The cofinite topology.

   Choose five (six*) of these topologies, and determine which are finer/coarser than the others. (You don't need to prove that one is inside another, but you should provide examples to show that one is NOT inside another.)

5. (*) Let $X$ be an infinite set equipped with the cofinite topology. For an arbitrary subset $Y \subset X$ describe: the closure $\overline{Y}$, the interior Int $Y$, the boundary $\partial Y$, and the subspace topology on $Y$. Treat the case of $Y$ finite and $Y$ infinite separately.

6. Let $\mathcal{S}\text{et}_*$ be the category of *pointed sets*. An object is a pair $(X, x)$ of a set $X$ and an element $x \in X$, the *special point*. Then $\mathrm{Mor}((X,x),(Y,y)) = \{f\colon X \to Y \mid f(x) = y\}$ is the set of all functions which send the special point to the special point. Composition is the usual composition of functions.

   (a) Prove that $\mathcal{S}\text{et}_*$ is a category. (Hint: you can use the fact that $\mathcal{S}\text{et}$ is a category! Really, I just want 2 lines here...)
   (b) Is there an initial object in $\mathcal{S}\text{et}_*$? Is there a final object in $\mathcal{S}\text{et}_*$?

7. Is it a category? If yes, just say so. If no, explain why not.

   (a) The objects are topological spaces; the morphisms are constant functions; composition is composition of functions.

(b) The objects are topological spaces;

$$\mathrm{Mor}(X,Y) = \{ \text{ pairs } (f,g) \text{ of continuous functions } f,g\colon X \to Y\};$$

composition is $(f,g) \circ (h,k) = (f \circ h, g \circ k)$.

(c) (*) The objects are topological spaces;

$$\begin{aligned}\mathrm{Mor}(X,Y) = \{ & \text{ pairs } (f,g) \text{ of continuous functions } f,g\colon X \to Y \\ & \text{such that } f = g \text{ except at finitely many points}\};\end{aligned}$$

composition is $(f,g) \circ (h,k) = (f \circ h, g \circ k)$.