Math 431/531 (Topology), Fall 2015
HW 4

Starred problems are for 531 students, and are extra credit for 431 students. 531 students must LaTeX their solutions.

1. Exercise 9.8def(j*) from K.
2. Exercises 1, 2, 3, 5, 6, 12* from Munkres p152
3. Exercises 2, 3 from Munkres p158
4. Recall that the topologist's sine curve S is the subset of $\mathbb{R}^2$ which is

$$S = \{(0, y) \mid -1 \leq y \leq 1\} \cup \{(x, \sin(\frac{\pi}{x})) \mid 0 \leq x \leq 1\}.$$

   Classify each point as an n-cut point.

5. (a) (*) Let $X \subset \mathbb{R}^2$ be the union of the graphs of $y = \frac{x}{n}$ for all $n \in \mathbb{Z}$, $n \geq 1$. What are the connected components of $X \setminus 0$? Is X connected? Is X locally connected?
   (b) (*) Now let $Y = X \cup \{(x, 0)\}$, which adds in the graph of $y = 0$. What are the connected components of $Y \setminus 0$? Is Y connected? Is Y locally connected?
6. (a) What is a 0-cut point? What is a local 0-cut point (be careful)?
   (b) Prove that a homeomorphism sends an n-cut point to an n-cut point, for any $n \geq 0$.
   (c) (*) Prove that a homeomorphism sends a local n-cut point to a local n-cut point, for any $n \geq 0$.
   (d) (*) What can you say about cut points in relation to continuous functions which need not be homeomorphisms? Can a 2-cut point be sent to a 3-cut point? Vice versa?