Math 431/531 (Topology), Fall 2015
HW 6

Starred problems are for 531 students, and are extra credit for 431 students. 531 students must LaTeX their solutions.

1. Exercise 6.2abcd, 6.6l* (with a non-discrete example too).
2. Exercise 1, 2 from Munkres p118.
3. Exercise 3, 4* (only the product and box topologies) from Munkres p126.
4. Define the infinite coproduct of topological spaces (with indexing set J). Prove that it has the desired universal property.
5. (a) Let X be a topological space, and U_α, $\alpha \in J$, be a collection of subsets which are all open and closed, which cover X, and for which $U_\alpha \cap U_\beta = \emptyset$ for each $\alpha, \beta \in J$. Prove that X is homeomorphic to the disjoint union of the U_α.
   (b) Is X always homeomorphic to the disjoint union of its connected components?
6. Let X be a topological space, and Z be an arbitrary disjoint union (possibly infinite) of copies of X. Show that Z is homeomorphic to $X \times Y$ for some space Y. What topology does Y have?
7. Let X and Y be infinite sets with the cofinite topology.
   (a) Describe the topology on $X \coprod Y$. (That is, describe the opens or closeds in some explicit way that is not just restating the definition of the coproduct.) Is it the cofinite topology?
   (b) Describe the topology on $X \times Y$. Is it the cofinite topology?
8. In each of the following categories, for two arbitrary objects X and Y, does the coproduct $X \coprod Y$ necessarily exist? If it does exist, what is it? Same question for the product $X \times Y$.
   (a) $\mathcal{S}\text{et}_*$, the category of pointed sets. (Recall: an object is a pair (S, s) of a set S with a special element $s \in S$. A morphism from (S, s) to (T, t) is a function which sends s to t.)
   (b) $(\mathbb{Z}, \leq)$, the order category. (Recall: an object is an integer n. There is exactly one morphism $n \to m$ if $n \leq m$, and no morphisms otherwise.)
   (c) (*) The category of finite dimensional vector spaces over the field $\mathbb{R}$.
   (d) (*) $\mathcal{S}\text{et}_*^{\text{even}}$. This is the category, defined exactly like $\mathcal{S}\text{et}_*$, except that the objects are finite sets of even size.