Exercises on Compactness + Integration

(1.) (These are proven in Bump Ch 1.) $\int f(ghg^{-1})dh = \delta(g)\int f(h)dh$ for left Haar measure dh.

Show δ is a homomorphism + continuous. Show $\delta(g)dg$ is a right Haar measure.

Show that $g \longmapsto g^{-1}$ is an isometry if G compact.

(2.) Finish the example on Bump p3 and show the right Haar measure is $x^{-1}dxdy$.

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3 Bump 1.1 4. Bump 1.2.

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(5) (Shown in Bump Prop 2.10) Prove that when V is irred, then $\mathcal{M}_V \cong V \otimes V^*$, as a G-bimodule

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6 Bump 2.1 7. Bump 2.2 8. Bump 2.3

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9. Bump 4.1 10 Bump 4.2

11. Explain why the theory of the Fourier expansion of a function in $L^2(S^1)$ is an instance of the Schur Orthonormality and Peter-Weyl theorems. Be precise!

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12. $G = (\mathbb{R}, +)$. a) Find the right and left Haar measures for G. Do they agree?

b) Show that reps of G are not semisimple by finding (reps with a) a subrep which is not a summand.