Exercises    Qtr 2 Wk 1

(1) Prove – $L$ simple $\Rightarrow$ the only 1D repn is trivial.

$\boxed{2.}$ Def: We say that $L$ is a central extension of $K$ if there is a s.e.s.

$0 \to I \to L \to K \to 0$ where $I \subset Z(L)$.

Prove that $L$ is nilpotent $\iff$ $L$ is an iterated central extension of abelian lie algebras

$\boxed{3.}$ Find the radical of $\mathfrak{p}_{m,n} = \begin{array}{c} \\ m \\ n \end{array}\!\!\begin{array}{c} \begin{array}{cc} m & n \end{array} \\ \left(\begin{array}{c|c} * & * \\ \hline 0 & * \end{array}\right) \end{array} \subset \mathfrak{gl}_{m+n}$. What is $[\mathfrak{p},\mathfrak{p}]$?

Is $\mathfrak{p}$ solvable? nilpotent?

4. Find the radical of $L = \begin{array}{c} \\ n \\ 1 \end{array}\!\!\begin{array}{c} \begin{array}{cc} n & 1 \end{array} \\ \left(\begin{array}{c|c} * & * \\ \hline 0 & 0 \end{array}\right) \end{array} \subset \mathfrak{gl}_{n+1}$. What is $[L,L]$?

Is $L$ solvable? nilpotent?

$\boxed{5.}$ Find the radical of $\mathrm{Heis} = \begin{array}{c} \\ 1 \\ n \\ 1 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & n & 1 \end{array} \\ \left(\begin{array}{c|c|c} 0 & * & * \\ \hline 0 & 0 & * \\ \hline 0 & 0 & 0 \end{array}\right) \end{array} \subset \mathfrak{gl}_{n+2}$. What is $[\mathrm{Heis},\mathrm{Heis}]$?

Is it solvable? Nilpotent?

$\boxed{6.}$ a) Let $W, V$ be fd. v.s. and $\varphi : \Lambda^2 V \to W$ a linear map. Prove that

$L_\varphi = V \oplus W$  $[v_1,v_2] = \varphi(v_1 \wedge v_2)$  $[v,w] = 0$  $[w_1,w_2] = 0$

defines a lie algebra?

b) Solvable? Nilpotent? What is $[L,L]$?

c) Show that $L_\varphi \cong L_\psi \iff \exists\, g \in GL(V)$ $h \in GL(W)$ s.t. $\psi = (h^{-1})\varphi(\Lambda^2 g$

d) Compute $\dim \mathrm{Hom}_{\mathbb{C}}(\Lambda^2 V, W)$, $\dim GL(V)$, $\dim GL(W)$. Deduce that there are LOTS of nonisomorphic Lie algebras of this form, when $\dim V$, $\dim W$ are large.

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Humphreys  Ch 2 # $\boxed{5}$, 6, 7    Ch 3 # 2. Why is this different from exercise 2 above?
(4), $\boxed{7}$, $\boxed{8}$