Hwk Qtr 2 Wk2

$\boxed{1}$ a) Inspired by the defns of invariance for a bilinear form for groups + Lie algebras, give a defn for any Hopf algebra H.

b) Deduce that an invariant form is the same data as a morphism $V \to V^*$ of H-modules.

$\boxed{2}$ Compute the Killing form for $\mathfrak{sl}(2;\mathbb{R})$ and $\mathfrak{su}(2)$. Deduce once again that these real Lie algs are not isomorphic. (What are the signatures?)

$\boxed{3.}$ $\boxed{a}$ Consider $\mathfrak{sl}_4 \circlearrowright \Lambda^2\mathbb{C}^4$. Does this repn have an invariant form?

ok to skip

b) For which $\mathfrak{sl}_n$ does $\mathbb{C}^n$ have an invariant form?

— " — and which k — " — $S^k\mathbb{C}^n$ — " — ?

— " — $\Lambda^k\mathbb{C}^n$ — " — ?

(4.) Given that every fd. repn of L simple is comp. red., prove the same for L semisimple.

$\boxed{5.}$ a) Suppose $z \in Z(A)$ for an assoc alg A, and $z \circlearrowright V, W$ irreducible repns by two distinct eigenvalues. Prove that $\mathrm{Ext}^1(V,W) = 0$.

b) Now V, W not nec. irreducible, z acts by generalized evalues on V, W, w/ distinct evalues. Prove $\mathrm{Ext}^1(V,W) = 0$.

Humphreys Ch 5 # 1, 2, 3, $\boxed{4}$, $\boxed{5}$, 7.

Ch 6 # $\boxed{1}$, $\boxed{3}$, $\boxed{7}$