NAME ____

Astronomy 321
Final
March 24, 2023

Question 1

The following Table contains data for Main Sequence stars. Except for the Spectral classes and T_{eff}, which is in Kelvin, all quantities are in Solar units ($M_\odot$, $L_\odot$, and $R_\odot$).

a. Find scaling laws for $R(M_*)$, $L(M_*)$, and $T_{eff}(M_*)$.

b. Write the relations for $L_*(T_{eff})$, $R_*(T_{eff})$, and $M_*(T_{eff})$.

c. How closely can the relationships be described as power laws?

d. Are there changes in slope of the scaling relations as functions of the stellar mass M_*? If so what might they mean? (That is, do *kinks* in plots for the various stellar properties as functions of M_* have physical meaning? If so, what do you think causes the kinks?)

Main-Sequence Stars (Luminosity Class V)

Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
O5	42000	499000	13.4	60	−9.51	−4.40	−5.1	−1.19	−0.33
O6	39500	324000	12.2	37	−9.04	−3.93	−5.1	−1.17	−0.33
O7	37500	216000	11.0	—	−8.60	−3.68	−4.9	−1.15	−0.32
O8	35800	147000	10.0	23	−8.18	−3.54	−4.6	−1.14	−0.32
B0	30000	32500	6.7	17.5	−6.54	−3.16	−3.4	−1.08	−0.30
B1	25400	9950	5.2	—	−5.26	−2.70	−2.6	−0.95	−0.26
B2	20900	2920	4.1	—	−3.92	−2.35	−1.6	−0.84	−0.24
B3	18800	1580	3.8	7.6	−3.26	−1.94	−1.3	−0.71	−0.20
B5	15200	480	3.2	5.9	−1.96	−1.46	−0.5	−0.58	−0.17
B6	13700	272	2.9	—	−1.35	−1.21	−0.1	−0.50	−0.15
B7	12500	160	2.7	—	−0.77	−1.02	+0.3	−0.43	−0.13
B8	11400	96.7	2.5	3.8	−0.22	−0.80	+0.6	−0.34	−0.11
B9	10500	60.7	2.3	—	+0.28	−0.51	+0.8	−0.20	−0.07
A0	9800	39.4	2.2	2.9	+0.75	−0.30	+1.1	−0.02	−0.02
A1	9400	30.3	2.1	—	+1.04	−0.23	+1.3	+0.02	+0.01
A2	9020	23.6	2.0	—	+1.31	−0.20	+1.5	+0.05	+0.05
A5	8190	12.3	1.8	2.0	+2.02	−0.15	+2.2	+0.10	+0.15
A8	7600	7.13	1.5	—	+2.61	−0.10	+2.7	+0.09	+0.25
F0	7300	5.21	1.4	1.6	+2.95	−0.09	+3.0	+0.03	+0.30
F2	7050	3.89	1.3	—	+3.27	−0.11	+3.4	+0.00	+0.35
F5	6650	2.56	1.2	1.4	+3.72	−0.14	+3.9	−0.02	+0.44
F8	6250	1.68	1.1	—	+4.18	−0.16	+4.3	+0.02	+0.52

Appendix: Stellar Data

Main-Sequence Stars (Luminosity Class V)

Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
G0	5940	1.25	1.06	1.05	+4.50	−0.18	+4.7	+0.06	+0.58
G2	5790	1.07	1.03	—	+4.66	−0.20	+4.9	+0.12	+0.63
Sun[a]	5777	1.00	1.00	1.00	+4.74	−0.08	+4.82	+0.195	+0.650
G8	5310	0.656	0.96	—	+5.20	−0.40	+5.6	+0.30	+0.74
K0	5150	0.552	0.93	0.79	+5.39	−0.31	+5.7	+0.45	+0.81
K1	4990	0.461	0.91	—	+5.58	−0.37	+6.0	+0.54	+0.86
K3	4690	0.318	0.86	—	+5.98	−0.50	+6.5	+0.80	+0.96
K4	4540	0.263	0.83	—	+6.19	−0.55	+6.7	—	+1.05
K5	4410	0.216	0.80	0.67	+6.40	−0.72	+7.1	+0.98	+1.15
K7	4150	0.145	0.74	—	+6.84	−1.01	+7.8	+1.21	+1.33
M0	3840	0.077	0.63	0.51	+7.52	−1.38	+8.9	+1.22	+1.40
M1	3660	0.050	0.56	—	+7.99	−1.62	+9.6	+1.21	+1.46
M2	3520	0.032	0.48	0.40	+8.47	−1.89	+10.4	+1.18	+1.49
M3	3400	0.020	0.41	—	+8.97	−2.15	+11.1	+1.16	+1.51
M4	3290	0.013	0.35	—	+9.49	−2.38	+11.9	+1.15	+1.54
M5	3170	0.0076	0.29	0.21	+10.1	−2.73	+12.8	+1.24	+1.64
M6	3030	0.0044	0.24	—	+10.6	−3.21	+13.8	+1.32	+1.73
M7	2860	0.0025	0.20	—	+11.3	−3.46	+14.7	+1.40	+1.80

[a]Values adopted in this text.

Question 2

a. What is meant by *hydrostatic equilibrium*? What is meant by *thermal equilibrium*?

b. Explain how we know that Main Sequence stars are in rough hydrostatic and thermal equilibrium.

c. Calculate the timescales (ignoring pressure) on which a deviation from *hydrostatic equilibrium* is punished for the Sun, a $M_* = 1.4M_\odot$ CO white dwarf of radius 1,000 km, and $1.4M_\odot$ neutron star of radius 10 km. Derive the expression for the timescale using dimensional arguments.

d. Why doesn't the Sun collapse or disrupt if it suffers small perturbations to its radius?

Question 3

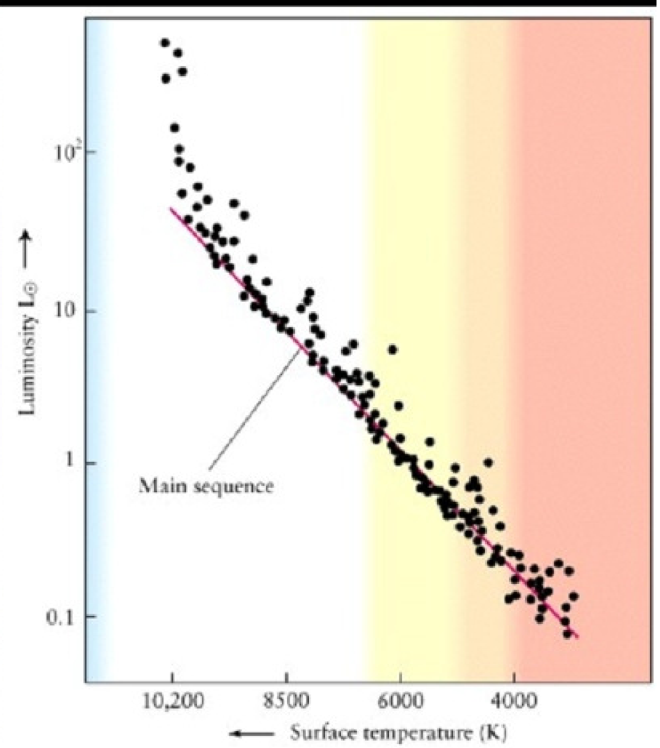
The figure shows the HR diagram for the Pleiades, a cluster of stars in the Milky Way galaxy. Assume that the stars in the Pleiades became stars at the same time, the stars in the Pleiades are all at the same distance from the Earth, and that the stars in the Pleiades are distributed in mass according to the Salpeter Initial Mass Function.

a. What is the most massive star left on the Main Sequence in the Pleiades. What is the least massive star seen in the Pleiades? Assume that the mass-luminosity relation is $L_* \propto M_*^4$.

b. What is the approximate age of the Pleiades? Refer to this as the Main Sequence turn-off age.

c. Is the Main Sequence turn-off age consistent with the observed appearance of the Main Sequence of the Pleiades? Explain your answer.

The Pleiades and Their HR Diagram



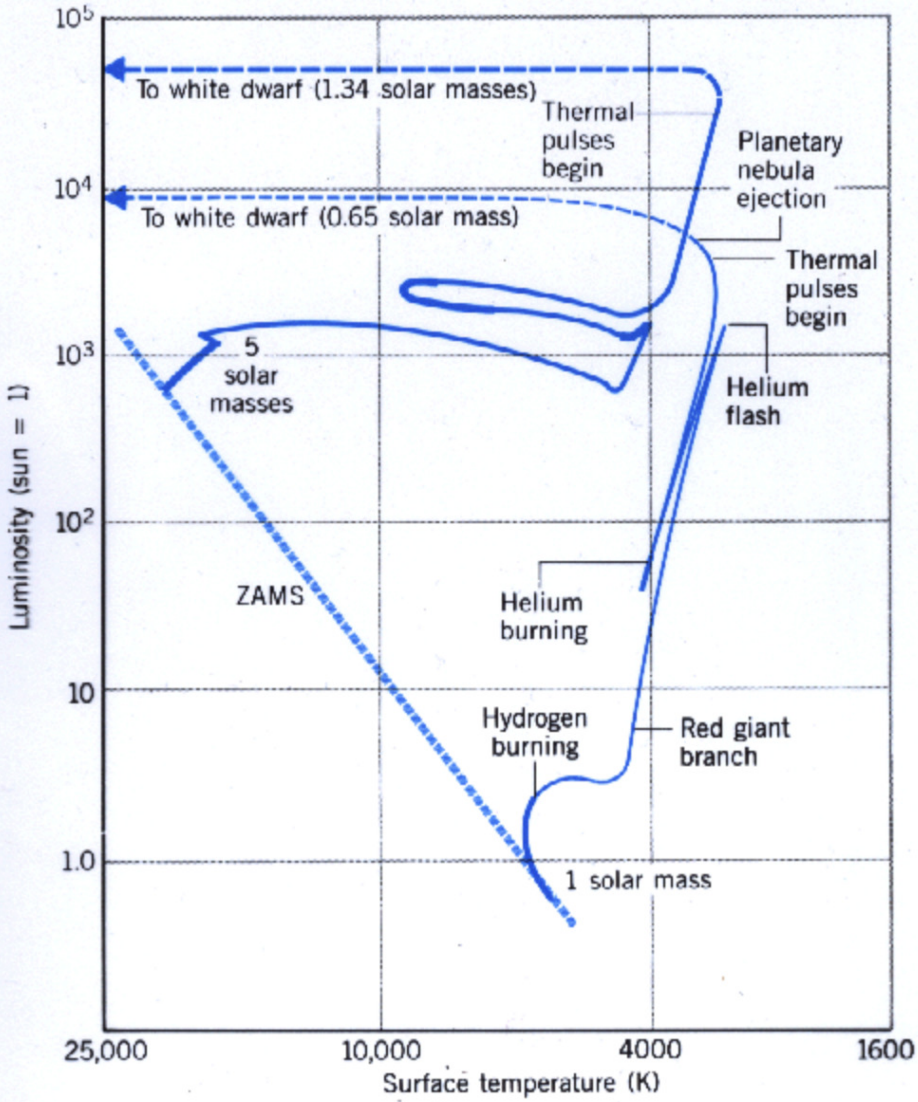
Question 4

a. The evolutionary track for the Sun is shown on the HR diagram. Starting with the Sun on the ZAMS. Label each stage of evolution (if not marked) give the rough time spent in the long-lived phases of evolution. For each stage of evolution (when appropriate), state how and where the Sun generates the energy needed to maintain thermal equilibrium.

b. By the end of the main sequence lifetime of the Sun, it has a helium core of mass ~ 0.1 $M_\odot$. Show that the central density ρ_c and pressure P_c of this core are related by $P_c = 0.36\, G M_c^{2/3} \rho_c^{4/3}$, with M_c the mass of the helium core. Assume that the pressure at the surface of the core is small compared to the central pressure.

c. At the temperature at which helium ignites, $T_{ign} \sim 10^8$ K, find the core density ρ_c. Assume that the ideal gas law is valid. Is the assumption of an ideal gas valid for the electrons? for the helium nuclei?

d. Why doesn't the track of a 5 $M_\odot$ star show the discontinuous drop to the horizontal branch?



Theoretical evolutionary tracks off the ZAMS for the stars of 1 and 5 solar masses.

Question 5

SN 1987a is, arguably, the most important observational result in stellar physics over the past 50 years.

a. Explain why SN 1987a could be the most important obsevational result in stellar physics over the past 50 years. Be sure to cite relevant observational results which support your answer.

b. Estimate the total energy release in Type II SN and compare your estimate to the energy output of Type Ia SN.

Question 6

A star is supported by radiation pressure so that its equation-of-state is given by

$$P = \frac{1}{3} a T^4 \tag{1}$$

where P is the pressure, a is the radiation constant, and T is the temperature.

a. If the star is in hydrostatic and radiative equilibrium, show that its luminosity is given by

$$L_{Edd} = \frac{4\pi c G M_*}{\kappa}, \tag{2}$$

where c is the speed of light, G is the gravitational constant, M_* is the mass of the star, and κ is the opacity in units of $cm^2\ g^{-1}$. The above luminosity, L_{Edd}, is known as the *Eddington Luminosity*.

b. A star has mass $M_* = 50\ M_\odot$, $L_* = 6 \times 10^5\ L_\odot$, and $\kappa = 0.4\ cm^2\ g^{-1}$. Calculate L_{Edd} for this star. How luminous is this star as a fraction of the Eddington luminosity, L_{Edd}?

c. What would happen if $L_* > L_{Edd}$?