Astronomy 321

Homework 3

Due: 2023/03/03

10 Calculate the Jeans length for the giant molecular cloud in Example 2.1 of "The Interstellar Medium and Star Formation."

13 **(a)** By using the ideal gas law, calculate $|dP/dr| \approx |\Delta P/\Delta r| \sim P_c/R_J$ at the beginning of the collapse of a giant molecular cloud, where P_c is an approximate value for the central pressure of the cloud. Assume that $P = 0$ at the edge of the molecular cloud and take its mass and radius to be the Jeans values found in Example 2.1 of "The Interstellar Medium and Star Formation" and in Problem10. You should also the cloud temperature and density given in Example 2.1.

(b) Show that, given the accuracy of our crude estimates, $|dP/dr|$ found in part (a) is comparable to (i.e., within an order of magnitude of) $GM_r\rho/r^2$, as required for quasi-hydrostatic equilibrium.

(c) Show that as long as the collapse remains isothermal, the contribution of dP/dr in the below equation continues to decrease relative to $GM_r\rho/r^2$, supporting the assumption made in Eq. (19) that dP/dr can be neglected once free-fall collapse begins.

$$\rho\frac{d^2r}{dt^2} = -G\frac{M_r\rho}{r^2} - \frac{dP}{dr}.$$

$$\frac{d^2r}{dt^2} = -G\frac{M_r}{r^2}. \qquad (19)$$

11 Show that the Jeans mass (Eq. 14) can also be written in the form

$$M_J \simeq \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2} \tag{14}$$

$$M_J = \frac{c_J v_T^4}{P_0^{1/2} G^{3/2}}$$

where the isothermal sound speed, v_T, is given by Eq. (18), P_0 is the pressure associated with the density ρ_0 and temperature T, and $c_J \simeq 5.46$ is a dimensionless constant.

$$v_T \equiv \sqrt{kT/\mu m_H} \tag{18}$$

9 **(a)** Assume an initial mass function of the form given in Eq. (5). If $x = 0.8$, calculate the ratio of the number of stars that are formed in the mass range between 2 $M_\odot$ and 3 $M_\odot$ to those formed with masses between 10 $M_\odot$ and 11 $M_\odot$.

$$\xi(M) = \frac{dN}{dM} = CM^{-(1+x)} \tag{5}$$

(b) Beginning with the initial mass function and using the mass–luminosity relation for main-sequence stars, derive an expression for the number of main-sequence stars formed per unit luminosity interval, dN/dL.

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha$$

(c) If $x = 0.8$ and $\alpha = 4$, calculate the ratio of the number of stars that are formed with main-sequence luminosities between 2 $L_\odot$ and 3 $L_\odot$ to the number of those formed with luminosities between 10 $L_\odot$ and 11 $L_\odot$.

(d) Compare your answers in parts (a) and (c), and explain the results in terms of the physical characteristics of stars along the main sequence.