Astronomy 321
Test 2
Due: Friday, February 24, 2023

Question 1

Using dimensional analysis:

a. find how the temperature of a main sequence star scales with its mass and radius when its internal pressure is given by the perfect gas law;

b. find how the temperature of a main sequence star scales with its mass and radius when its internal pressure is dominated by radiation pressure.

c. The ratio of gas pressure to radiation pressure in the core of the Sun is about 10^4. Using this result, estimate the mass of a Main Sequence star where radiation pressure is comparable to gas pressure in the core of the star. Assume that up to this limiting mass, the gas pressure dominates radiation pressure in the core of the star.

Question 2

a. The first step in the proton-proton chain is the p-p reaction. Fill in the missing parts of the p-p reaction,

$$p + p \rightarrow d + ? + ? + ? \tag{1}$$

What conservation laws did you apply to fill in the reaction?

b. The primary obstacle to nuclear fusion is the electrostatic repulsion between the fusing nuclei. Estimate the energy needed to overcome the electrostatic repulsion of the protons. How many times larger is this energy than $0.5 m_p v_{rms}^2$ for a gas with temperature 1.4×10^7 K. Here, m_p is the proton mass and v_{rms} is the root mean square velocity.

c. Estimate the number of cycles of the proton-proton chain which must take place every second to account for the luminosity of the Sun.

d. Estimate the flux of neutrinos due to the proton-proton chain at the orbit of the Earth. The average distance from the Earth to the Sun is 1.5×10^{13} cm, the Astronomical Unit.

e. How long does it take a neutrino to reach the Earth from the core of the Sun? Roughly how long would it take a photon to diffuse to the surface of the Sun assuming electron scattering opacity. Roughly how long would it take a photon to diffuse to the surface of the Sun assuming bound-free and free-free opacity? The preceding explains why the difficult to detect neutrinos are used to probe the core of the Sun rather than the more plentiful and easily detected photons from the Sun.

Question 3

a. What is meant by *hydrostatic equilibrium*? What is meant by *thermal equilibrium*?

b. Explain why we know that Main Sequence stars are roughly in hydrostatic and thermal equilibrium.

c. Calculate the timescales on which a deviation from *hydrostatic equilibrium* is punished for the Sun, a $M_* = 1.2 M_\odot$ CO white dwarf of radius 1,000 km, and $1.4 M_\odot$ neutron star of radius 10 km, if pressure effects are ignored. Derive the expression for the timescale using dimensional arguments.

d. Why doesn't the Sun collapse or expand rapidly whenever it suffers small perturbations to its radius?

Question 4

a. Rewrite the radiation transport equation replacing the temperature gradient with the pressure gradient. Recall that the radiation energy density U_γ is three times the radiation pressure,

$$U_\gamma = 3P_\gamma = aT^4. \quad (2)$$

b. The relation found in Part a shows the acceleration produced by an amount of radiation L_* flowing out of a star. Using this relation find an expression for the L_* that leads to an outward pressure that exceeds the inward pull of gravity. This limiting L_* is referred to as the Eddington limit.

c. Evaluate the Eddington limit for a star of mass of one solar mass, 1 $M_\odot$ dominated by electron scattering, κ = 0.2(1+X).

d. Estimate the upper limit for Main Sequence stars using the mass-luminosity relationship or use the table for the properties or Main Sequence stars given with Test 1.

e. If we calculated the Eddington limit for the Sun, how would the limit change? Just describe in words, no need to derive anything.

Question 5:

a. Use dimensional arguments to find the mass-luminosity relation including κ for Main Sequence stars. What is the mass-luminosity relationship for stars dominated by electron scattering?

b. Use dimensional arguments to show that the mass-luminosity-radius relation for Main Sequence stars is

$$L_* \propto \frac{M_*^{5.5}}{R_*^{0.5}}, \tag{3}$$

for Kramer's opacity approximation, $\kappa \propto \rho T^{-3.5}$.

c. Eliminate R_* from the relationship in Part b.