NAME

Astronomy 321
Test 3
March 17, 2023

Do 4 out of the 5 following questions.

Question 1

a. Show using dimensional arguments that for electrons, quantum effects (degeneracy) become important when the density exceeds

$$\rho \propto \mu_e m_h \left(m_e k T\right)^{1.5} \hbar^{-3}. \tag{1}$$

Here ρ is the mass density, μ_e is the mean moleular weight per electron $\mu_e = 2/(1+\mathrm{X})$, m_h is the the hydrogen mass, m_e is the electron mass, $\hbar$ is Planck's constant, k is the Boltzmann constant, and T is the gas temperature. Evaluating exactly leads to $\rho > 10.6 \times (T/10^6\mathrm{K})^{1.5}$ gram per cubic centimeter.

b. The *Helium Flash* begins when the density is $\sim 10^5$ g cm^{-3} and T = 10^8 Kelvin. The outburst ends when degeneracy is lifted, that is, when the electrons become non-degenerate. Calculate the temperature when degeneracy is lifted.

c. Calculate the ratio of the energy generation rate at the time degeneracy is lifted to the energy generation rate at the onset of the flash. If you were unable to find the temperature at which degeneracy is lifted, use T = 4×10^8 Kelvin for your calculation of the energy generation rate.

Question 2

a. On an Hertzsprung-Russell diagram, sketch the evoltuionary track for the Sun starting with the Sun on the ZAMS. Label each stage of evolution.

b. Give the time the Sun spends in each long-lived phase of evolution.

c. For each stage of evolution (when appropriate), state how and where the Sun generates the energy needed to maintain thermal equilibrium.

d. What causes the abrupt change in the Sun when it moves up the AGB?

Question 3

a. Stars are categorized as low mass or high mass based on what criterion? What mass is the rough dividing line between high mass and low mass stars?

b. What are the rough upper and lower limiting masses for Main Sequence stars? Briefly, why do the limits arise?

c. Find the energy released during the normal nuclear burning lifetime of the Sun. Do the same for a 25 solar mass star.

Question 4

Assume the Milky Way galaxy contains 5×10^{10} $M_\odot$ of gas and $\sim 10^{11}$ stars that were formed with an initial mass function

$$\frac{dN}{dM} = aM^{-2.35}. \tag{2}$$

over the mass range 0.4 $M_\odot$ to 100 $M_\odot$.

a. Estimate the fraction of stars that form with mass $M_* > 8$ $M_\odot$, the lower limit for a star to undergo a Type II supernova outburst.

b. Assume that every supernova outburst distributes 0.05 $M_\odot$ of iron into the interstellar medium (ISM). What is the mean ISM mass abundance of iron in the Galaxy? State any further assumptions you make for your calculation. Compare your answer to the measured iron abundacne in the Sun $Z_\odot = 0.00177$. Explain why this shows that the Sun is a second generation star that was formed from pre-enriched ISM material.

Question 5

You will investigate white dwarf cooling in this question. The first law of thermodynamics is

$$dQ = dU + dW \tag{3}$$

where dQ is change in heat, dU is the change in internal energy and dW is the work. Assume that white dwarfs cool with constant radius, R_*, and that the internal temperature T can be approximated as $T = T_{eff}$ where T_{eff} is related to L_* and R_* as $L_* = 4\pi R_*^2 \sigma T_{eff}^4$.

a. Show that the first law of thermodynamics can be rewritten as

$$-L_* dt = -C_V M_* dT_{eff} \tag{4}$$

where dt is the change in time, C_V is the specific heat at constant volume, $C_V = 1.5(k/m)$ where m is average mass of a particle in the gas, and M_* is the mass of the white dwarf.

b. If the white dwarf cools from luminosity $L_\circ$ to a final luminoisty of L_f, show that

$$t = -\frac{M_* C_V}{(4\pi\sigma R_*^2)^{1/4} L_f^{3/4}} \left(1 - \left(\frac{L_f}{L_i}\right)^{3/4}\right) \tag{5}$$

c. Use the cooling time expression in Part b to estimate the time it takes for a 0.5 $M_\odot$ white dwarf, $R_* \sim 10^9$ cm, to cool from 0.01 $L_\odot$ to 0.0001 $L_\odot$.

d. What would be the consequence of observing a low luminosity, 0.5 $M_\odot$ white dwarf as described in Part c?