Astronomy 321
Homework 4
Due: Tuesday, 25 February 2020

19. Chapter 12, Problem 2
20. Chapter 12, Problem 6
21. Suppose that rotation cannot be ignored in a star.

a. Write down an expression for the equilibrium condition for material in the equatorial plane of the star. Assume that the star rotates about the z-axis and that the equatorial plane sits in the xy-plane.

b. A 1 $M_\odot$ star is observed to rotate with a period of 30 milliseconds. If this rapidly rotating star is stable (that is, the star does not fly apart due to the rotation), estimate the minimum mass density allowed for the star. Consider the centripetal acceleration on a test mass at the equator of the star to arrive at your estimate.

c. How does this mass density compare to the average mass density for the Sun, 1.4 g cm^{-3}?

22. A star of mass M_* and radius R_* has density profile $\rho(r) = \rho_\circ(1 - r/R_*)$.

a. Find $M(r)$ and express the total mass M_* in terms of R_* and $\rho_\circ$. How does this answer compare to that for an uniform density star and the estimate made in class?

b. Find $P(r)$ and express the central pressure in terms of M_* and R_*. How does this answer compare to that for an uniform density star and the estimate made in class?

c. Find $T(r)$ if the star is dominated by gas pressure.

23. A star has mass $M_* = 10M_\odot$ and is pure ^{12}C. The star has core temperature 600 million K compared to the Sun where the core temperature is 15 million K.

a. What is the mean particle mass in terms of the the mass of hydrogen?

b. Use the ideal gas law, the dimensional relationship between M_*, ρ_c, and radius R_*, and the virial theorem to find the scaling relationship between the stellar radius R_* with the total mass M_*, the mean particle mass, and the core temperature. Use the values for the Sun, find the radius of the star.

c. If the star's luminosity is 10 million $L_\odot$, find the star's effective temperature.

d. Suppose the star produces energy from

$$^{12}C +^{12} C \rightarrow^{24} Mg \quad (1)$$

The atomic weight of ^{12}C is 12 and that of ^{24}Mg is 23.985. What fraction of the mass of the star can be converted to energy?

e. How long does it take the star to use up 10% of its carbon?