Astronomy 321
Homework 3
Due: Friday, December 4, 2009

Question 1:

Estimate the duration of the horizontal branch phase in the evolution of a 1 $M_\odot$ star.

a. Compute the energy released in the Triple Alpha process, 3^4he $\rightarrow$ ^{12}C. The mass of the helium and carbon are 4.0026 and 12.000 a.m.u., respectively, where the atomic mass unit (a.m.u.) is $1.6606 \times 1-^{-27}$ kg.

b. Assume that at the beginning of the horizontal branch, 10 % of the original mass of the star is in the form of helium in the stellar core. Estimate the total energy released via the Triple Alpha process.

c. Assume that during the horizontal branch, $L_* = 100\ L_\odot$. If all of the luminosity is provided by the fusion of helium to carbon, how long will the horizontal branch phase last?

Question 2:

Most of the energy released in a Type II SN is in the form of neutrinos.

a. If a neutron star is produced in the core collapse with mass 1.4 $M_\odot$ and radius 10 km, estimate the mean density of nucleons. Find the mean free path for a neutrino inside the neutron star assuming that the neutrino cross-section is 10^{-42} cm^2.

b. How many seconds would it take a neutrino to random walk out of the neutron star? Recall that neutrinos travel at nearly the speed of light and for a random walk of N steps each of length l, the distance covered by the particle is $d = l\sqrt{\mathrm{N}}$.

c. 12 electron neutrinos from SN 1987a were detected by the Kamiokande neutrino detector in Japan. The detector consists of 3 kilo-tons of water surrounded by photomultiplier tubes. The protons in the water absorb an anti-neutrino from SN 1987a and then emit a positron. The positron travels with speed greater than c$/n$, where n is the index of refraction and so emits Cerenkov radiation which is detected by the photomultiplier tubes. Estimate the number of people on the Earth who could have seen a neutrino from SN 1987a by the same process. An eyeball is primarily water (about 10 g worth) and the population of the Earth was 5 billion in 1987.

Question 3:

Type Ia SNe are produced by CO white dwarfs near the Chandrasekhar limit.

a. Use the Virial Theorem to obtain an expression for the mean pressure inside a white dwarf of mass M and radius R.

b. Estimate the speed of sound in the white dwarf using the results of a. The speed of sound is given by

$$c_s = \sqrt{\gamma \frac{P}{\rho}}. \tag{1}$$

When the C ignites in the white dwarf, a *burning* front moves around the star igniting the C everywhere. The front moves at around the sound speed. How long does it take the *burning* front to traverse the star?

c. Calculate the energy released in the outburst (assume that carbon burns to nickel with a 0.1 % efficiency) under the assumption that the white dwarf is pure carbon. How does this energy compare to the gravitational energy of the white dwarf?

d. Gamma rays are produced in

$$^{56}\mathrm{Ni} \rightarrow ^{56}\mathrm{Co} \rightarrow ^{56}\mathrm{Fe} \tag{2}$$

which power the luminosity of the Type Ia SNe after the initial burst. The atomic weights of Ni and Fe are 55.942135 and 55.934941 a.m.u., respectively. Calculate the energy radiated in the optical range during the event. If the characteristic time scale for the decays are 8.8 days and 111 days, respectively, show that the luminosity is roughly 1 billion $L_\odot$.

Question 4:

Suppose we live in a Newtonian universe whose density is equal to the critical density (assuming $\Lambda = 0$).

a. What is the functional form of the scale factor $R(t)$ for given conditions at $t = t_\circ$ when $R(t_\circ) = 1$?

b. What is $t_\circ$ in terms of the Hubble constant?

c. in our Universe, $H_\circ$, the Hubble constant is 70 km s^{-1} Mega-parsec^{-1}. The oldest stars have ages on the order of 13 billion years (how are these ages estimated?). Are these ages consistent with a Newtonian universe that has the critical density?

Question 5:

Consider a close binary system with orbital period P, orbital separation a, and whose stars have mass M_p and M_c, where $M_p > M_c$. Treat the stars as point masses and assume that they are in circular orbits.

a. Derive the generalized Kepler's Third of Planetary Motion for this system.

b. In the frame which rotates with the binary system, write down an expression for equipotential surfaces. Assume that stars rotate synchronously with their orbital motion. Use xy coordinates where the stars lie on the y-axis.

c. Sketch several equipotential surfaces.

d. For given $q = M_p/M_c$, and P, find the location of the Inner Lagrange Point, L_1.

e. If $M_p = 0.8\ M_\odot$, $M_c = 0.2\ M_\odot$, and $r_c = r_\odot$, for what period P (and orbital size a) do we expect mass transfer to occur?