

Astr 321: Stellar Structure & Evolution

textbook: "An Introduction to Modern Astrophysics,"  
Bradly Carroll & Dale Ostlie

Material: Part II; chapters 7-18 (and some of 3-5)

## Basic Properties of Stars (chapters 7-9)

### (I) "Normal Stars"

- Bound by self-gravity; supported by pressure
- radiates energy as supplied by an internal energy source

### general implications

① if not rotating, non magnetic  $\Rightarrow$  only gravity acts and since gravity is a central force (spherically symmetric force field)  $\Rightarrow$  such stars are spherical

②  $\star$ s evolve: radiate  $\Rightarrow$  lose energy  $\Rightarrow$  need energy source (nuclear reactions  $\Rightarrow$  in cores  $\Rightarrow$  P changes  $\Rightarrow$  contracts: gravitational release  $\Rightarrow$   $R \downarrow$  to decrease  $V_g \Rightarrow$  evolution)

We will consider  $\star$  for a while  $\Rightarrow$  normal lifetime  $\Rightarrow$  end states (stellar remnants)

### (II) How do we study $\star$ s?

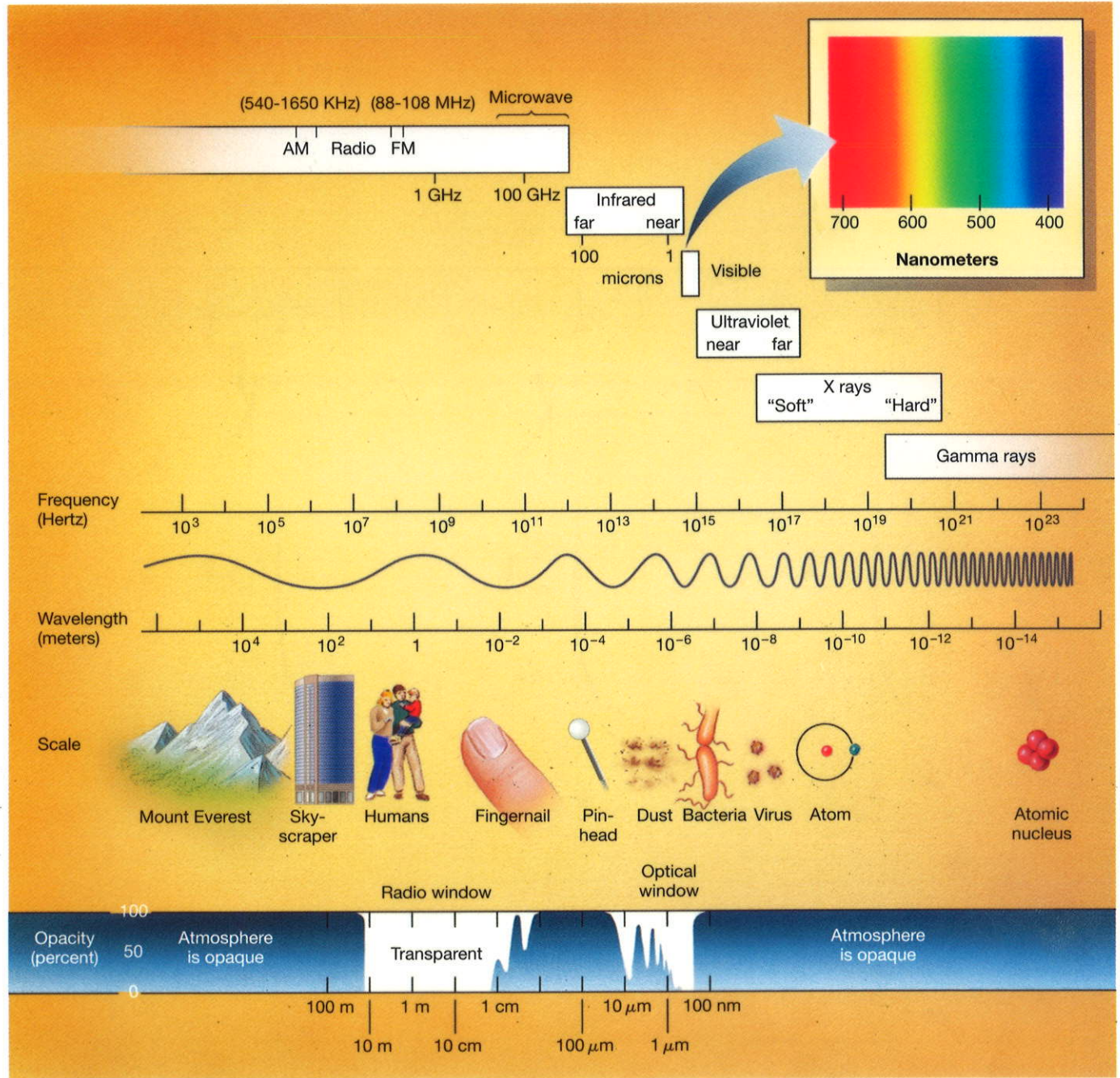
a) electromagnetic radiation

b) particle emission ( $\nu$ 's from  $\odot$  and SN)

c) gravitational radiation (binary pulsar)

Comment: all methods are passive; we observe we do not "experiment" and we obtain snapshots of  $\star$ s at single steps of their evolution (we do not follow single  $\star$ s thru their life cycles)

Figure 3.9 Electromagnetic Spectrum





(IV) What do we know about  $\Phi_s$ ?

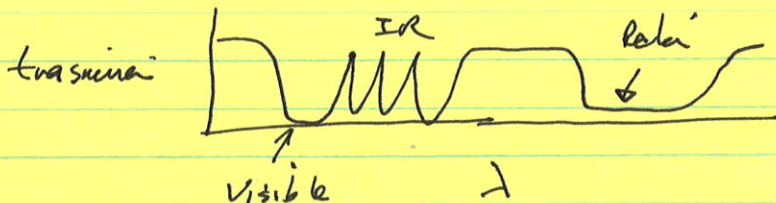
EM radiation:

- ① spectrum,  $I(\lambda) d\lambda$  or  $I(\nu) d\nu$
- ② polarization
- ③ temporal properties
- ④ power ("luminosity")

(i) Unfortunately, what we observe is the apparent brightness,  $I_{obs}$ , the light incident at the  $\oplus$  per unit area perpendicular to it, not the luminosity,  $L_*$ .

Ideally, we measure

$$I_{obs} = \int_0^{\infty} I_{obs}(\lambda) d\lambda \Rightarrow \text{we already see issues,}$$



$\Rightarrow$  we must get above the atmosphere to measure the total apparent brightness, bolometric brightness.

(ii)



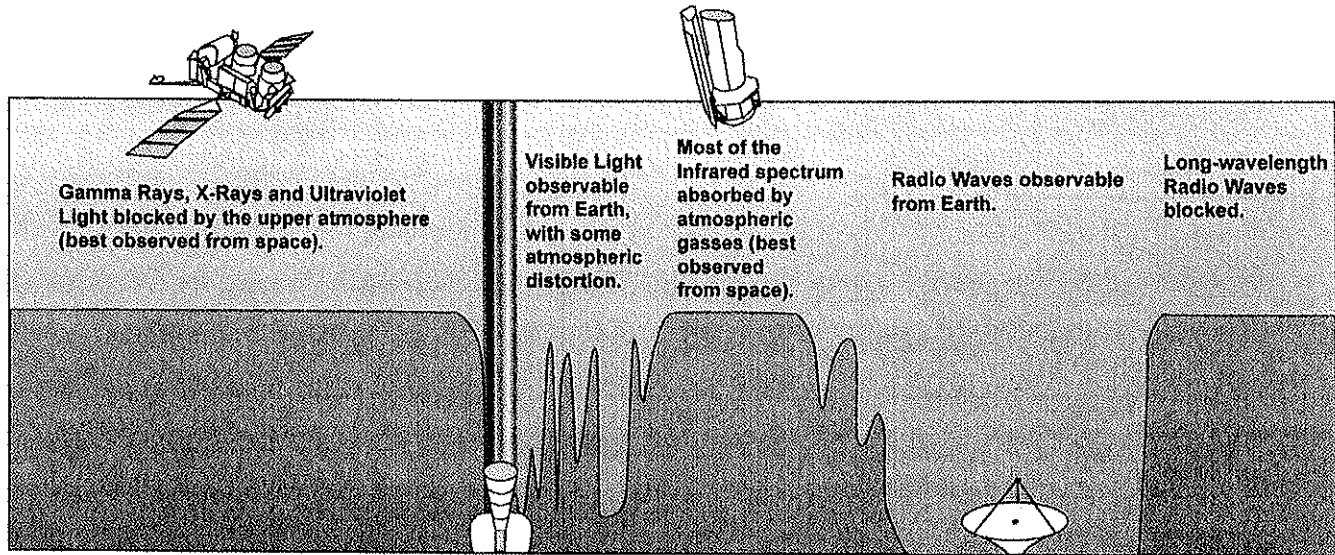
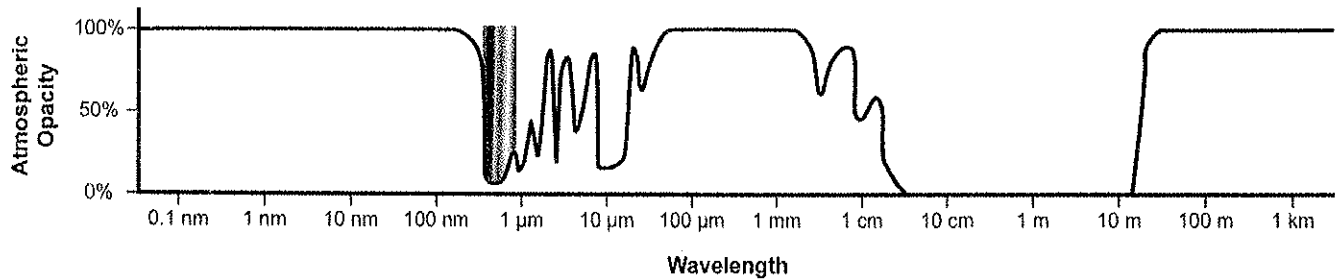
$$I_{obs} = \frac{\text{Luminosity}}{4\pi r^2} = \frac{L_*}{4\pi r^2}$$

inverse square law

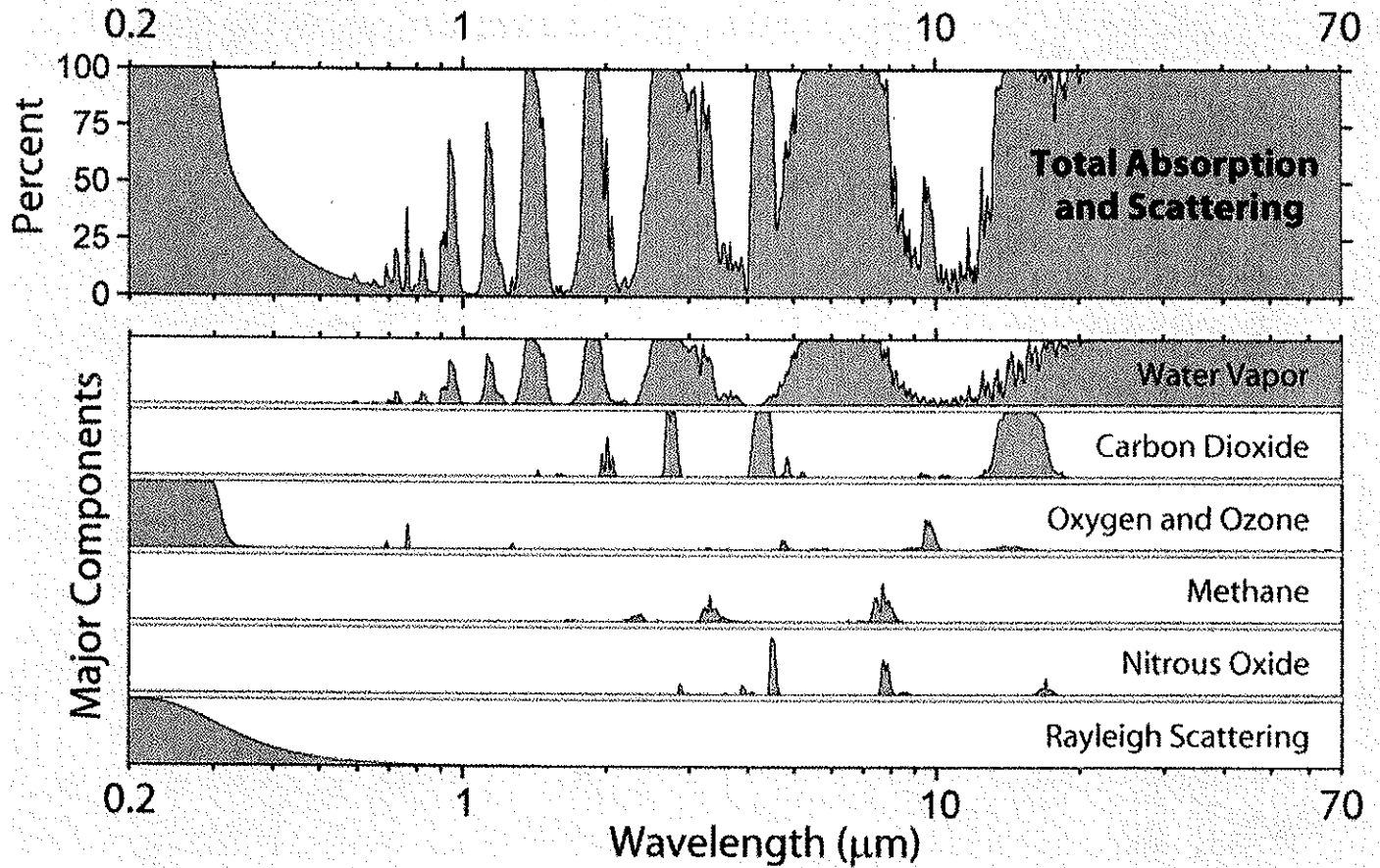
$$\Rightarrow I_{obs} = \int_0^{\infty} I(\lambda) d\lambda = \frac{L_*}{4\pi r^2} \Rightarrow L_* = 4\pi r^2 I_{obs}$$

to find intrinsic power, need  $r$ , distance &  $I_{obs}$  the bolometric brightness

characteristic  $L_*$  is  $L_{\odot} = 3.85 \times 10^{26} \text{ J s}^{-1}$ ; normal  $*$ s have a range of  $10^{-5} L_{\odot} \lesssim L_* \lesssim 10^5 L_{\odot}$

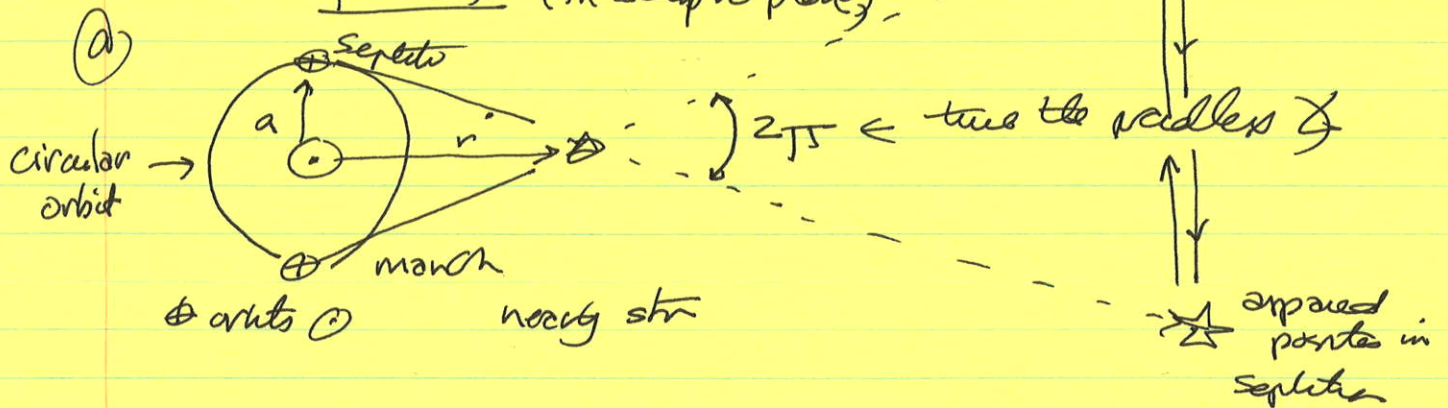


# Atmospheric Absorption Bands



there are only 2 "direct views" to hit r

parallax (in ecliptic plane)



r then follows from  $\tan \pi = \frac{a}{r}$

for small  $\pi \Rightarrow \tan \pi \approx \frac{\sin \pi}{\cos \pi} \approx \frac{\pi}{1 - \frac{1}{2}\pi^2} \approx \pi + \frac{1}{2}\pi^3$

$r \approx \frac{a}{\pi}$

outside of the ☉, closest ☆ is Proxima Centauri,

$r \approx 4.3 \text{ year}$

$1 \text{ year} = 9.46 \times 10^{15} \text{ m} \Rightarrow r \approx 4.3 \times 9.5 \times 10^{15} \approx 4.1 \times 10^{16} \text{ m}$

$a = 1.50 \times 10^{16} \text{ m} \Rightarrow \pi_{\alpha} = \frac{1.5 \times 10^{16}}{4.1 \times 10^{16}} \approx 3.7 \times 10^{-6} \text{ Radian}$

(i)  $2\pi \text{ Radian} = 360^\circ \Rightarrow 1 \text{ Radian} = \frac{360^\circ}{2\pi} = 57.3^\circ$

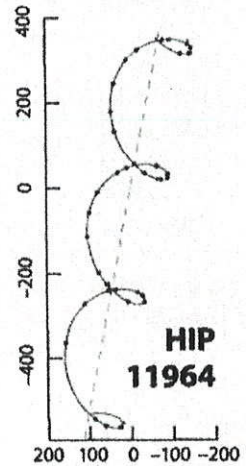
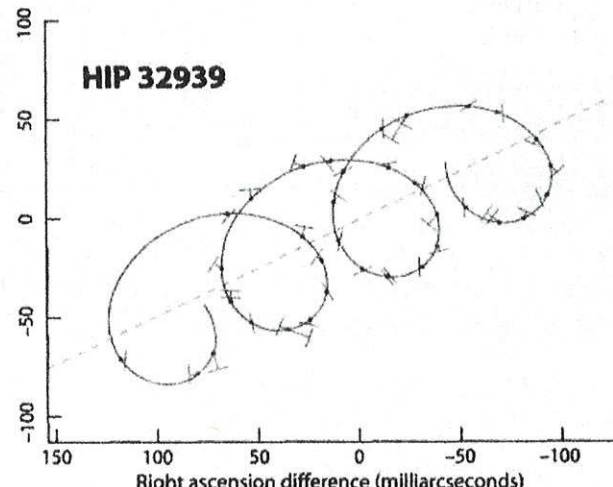
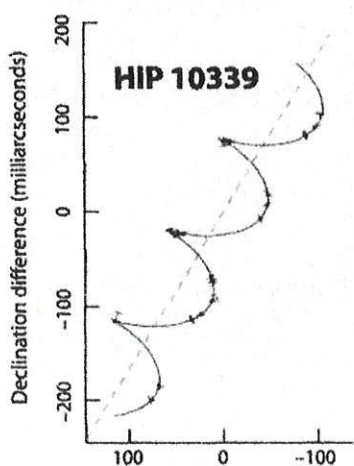
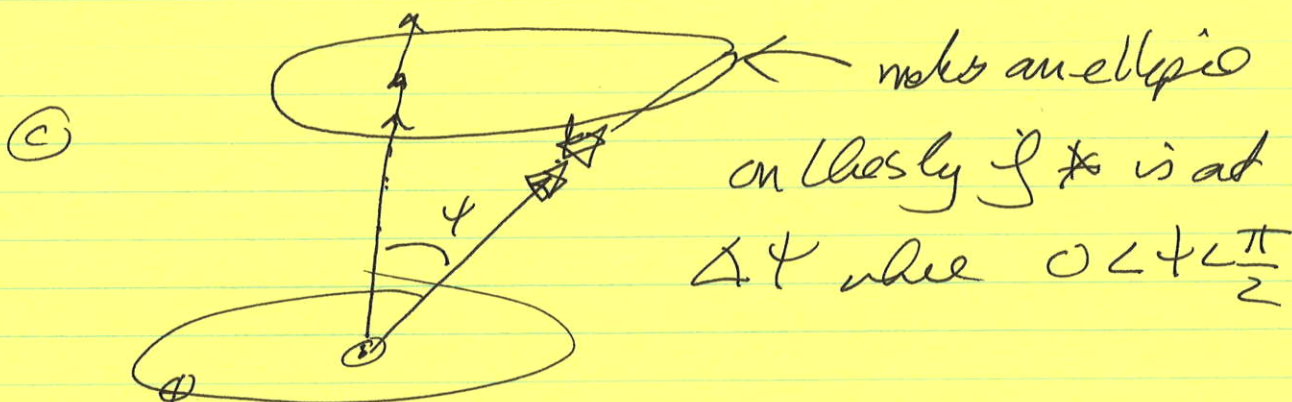
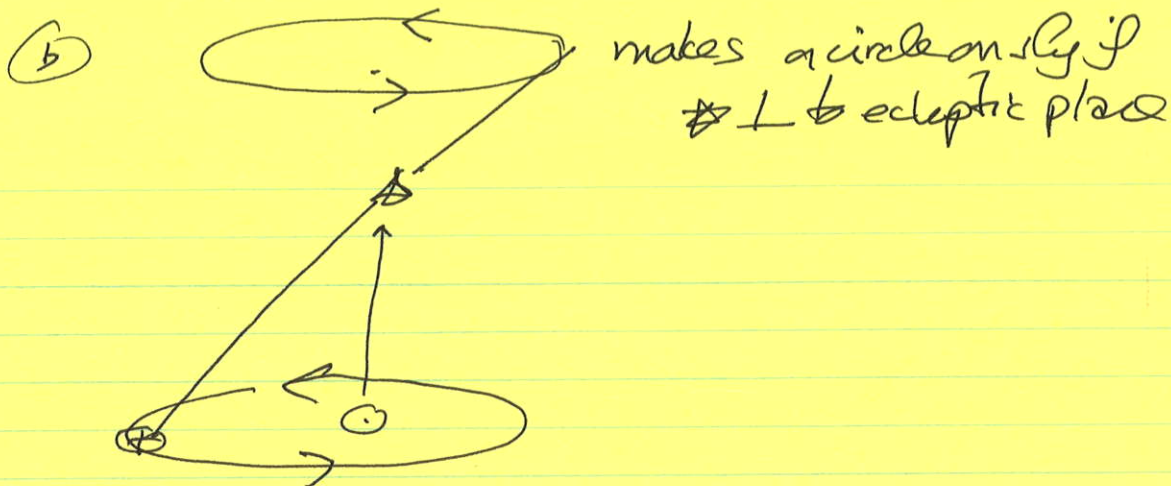
(ii)  $1^\circ = 60'$  &  $1' = 60'' \Rightarrow 1^\circ = 3600''$

$\Rightarrow \pi_{\alpha} \approx 0.76'' \ll 1^\circ \ll 1 \text{ Radian}$

def<sup>n</sup> 1 parsec is r for which  $\pi = 1''$

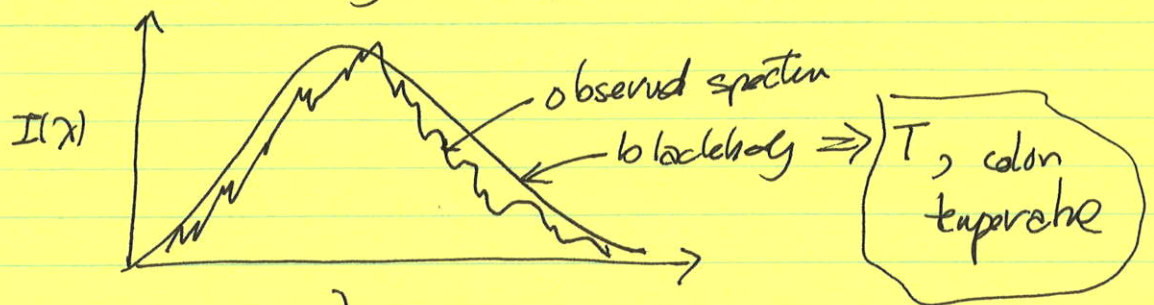
or  $r_p = 3.08 \times 10^{16} \text{ m} = 3.26 \text{ light years}$





The apparent paths of three stars across the sky during the three years of the Hipparcos mission. Each looping line shows the combination of parallax (an ellipse) and proper motion (a straight line) that best fits the data. The star's measured positions are shown by T-like intersections; these are often hidden under the dots, which mark their best-fit places on the line. Each curlicue in the 118,000-star database is different. From the Hipparcos Intermediate Data Web page.

(A) Can estimate the surface temperature of a star from  $I(\lambda)$  &  $L_*$  using an important result.



Stellar spectra are similar to black bodies,

$$B_{\lambda}(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}, \text{ Planck function}$$

$$h = 6.626 \times 10^{-34} \text{ J-s}$$

$$c = 2.998 \times 10^8 \text{ cm s}^{-1}$$

$$k = 1.381 \times 10^{-23} \text{ J K}^{-1}$$

a) Wien's law

$\lambda$  at which  $B(\lambda)$  peaks follows from

$$\frac{dB(\lambda)}{d\lambda} = \frac{-\frac{10hc^2}{\lambda^6}}{e^{hc/\lambda kT} - 1} - \frac{\frac{2hc^2}{\lambda^5}}{(e^{hc/\lambda kT} - 1)^2} \left( -\frac{hc}{\lambda^2 kT} e^{\frac{hc}{\lambda kT}} \right) = 0$$

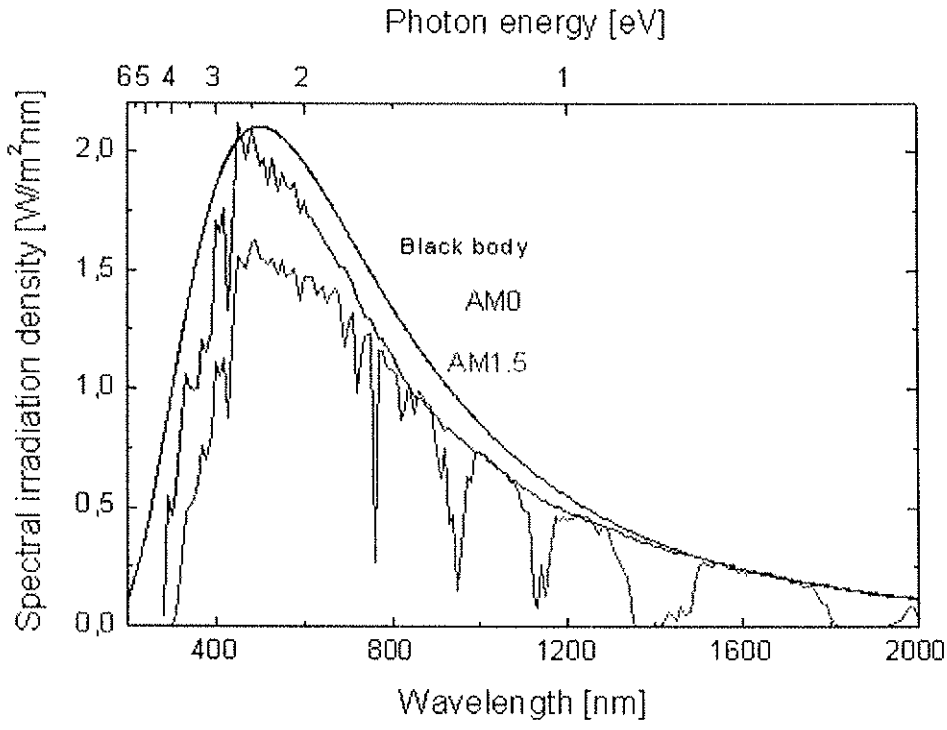
$$\frac{-10hc^2}{\lambda^6} (e^{\frac{hc}{\lambda kT}} - 1) + 2 \frac{h^2 c^3}{kT \lambda^7} e^{\frac{hc}{\lambda kT}} = 0$$

~~$$e^{\frac{hc}{\lambda kT}} \left[ \frac{2hc^3}{kT \lambda^7} - \frac{10hc^2}{\lambda^6} \right] = 0$$~~

$$\left( e^{\frac{hc}{\lambda kT}} - 1 \right) - \frac{1}{5} \frac{hc}{\lambda kT} e^{\frac{hc}{\lambda kT}} = 0$$

$$\Rightarrow \lambda_{\text{max}} T = 0.002898 \text{ m-K}$$

Wien's law



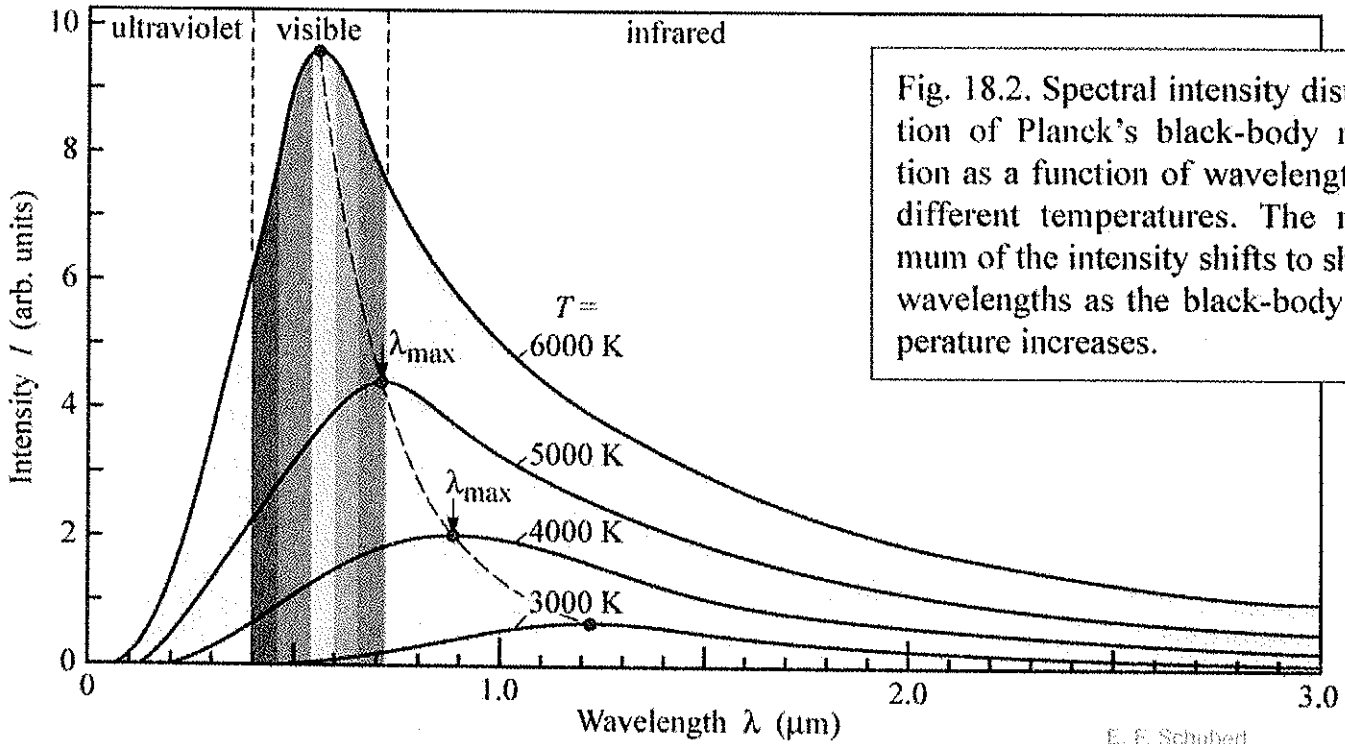


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

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*Light-Emitting Diodes* (Cambridge Univ. Press)  
[www.LightEmittingDiodes.org](http://www.LightEmittingDiodes.org)