

Convert to ν from λ

$$B_\nu(T) d\nu d\Omega = B_\lambda(T) d\lambda d\Omega$$

$$(i) B_\nu(T) = \frac{2hc^2 \left(\frac{\nu}{c}\right)^5}{e^{h\nu/kT} - 1} \quad \text{because } \nu\lambda = c \Rightarrow \frac{c}{\lambda} = \nu$$

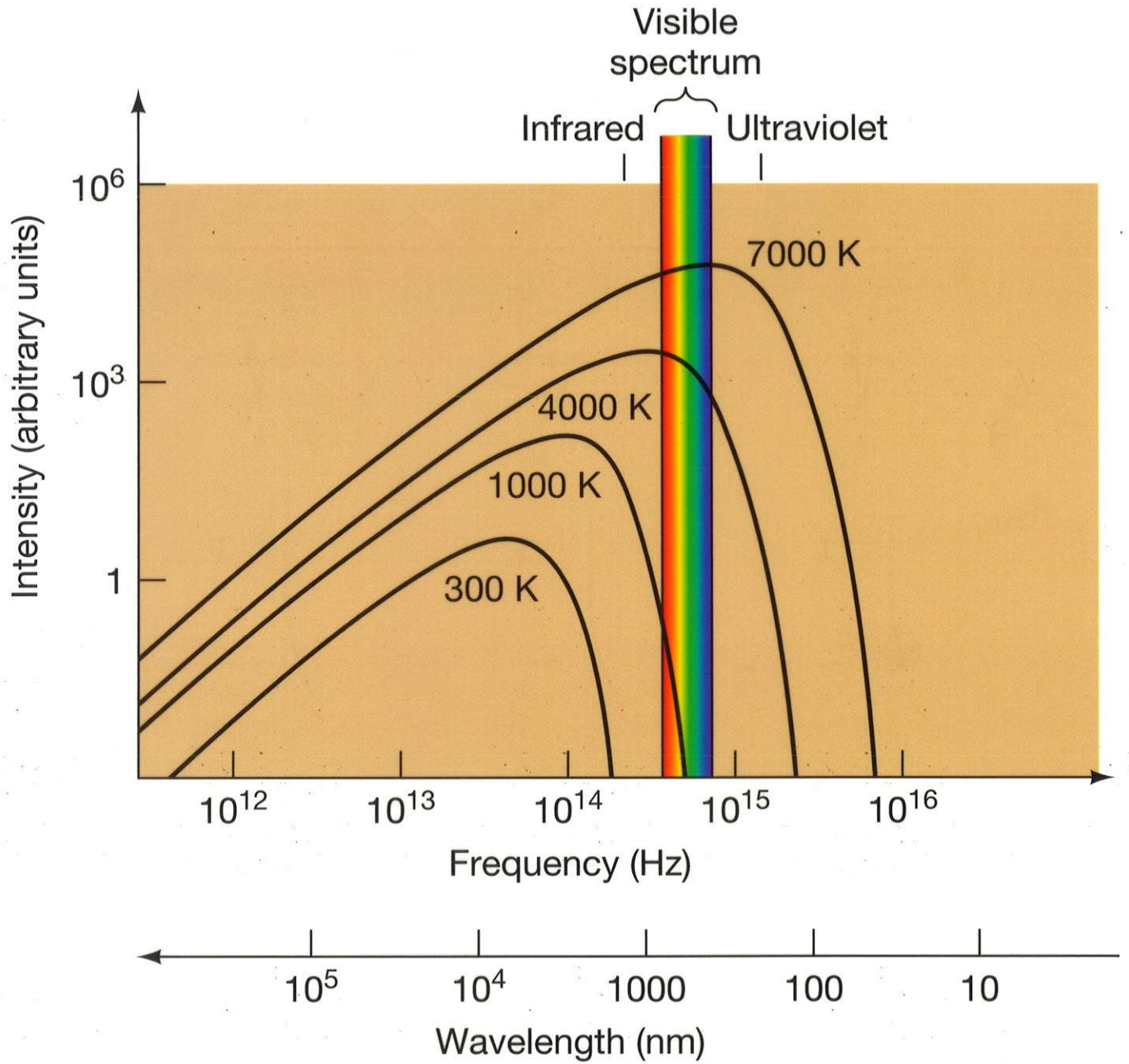
$$(ii) d\lambda = \left| -\frac{c}{\nu^2} \right| d\nu$$

$$\Rightarrow B_\nu(T) d\nu = \frac{2hc^2 \left(\frac{\nu}{c}\right)^5}{e^{h\nu/kT} - 1} \left(\frac{c}{\nu^2} d\nu \right)$$

$$B_\nu(T) d\nu = \left(\frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1} \right) d\nu$$

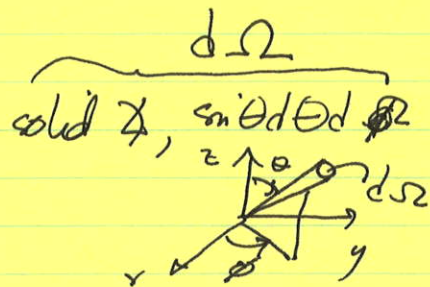
$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

Figure 3.11 Blackbody Spectrum



b) Stefan-Boltzmann Equation

$$f = \int_0^\infty B(\lambda) d\lambda d\Omega = \text{flux}$$



$$f_{SB} = \sigma T^4$$

$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

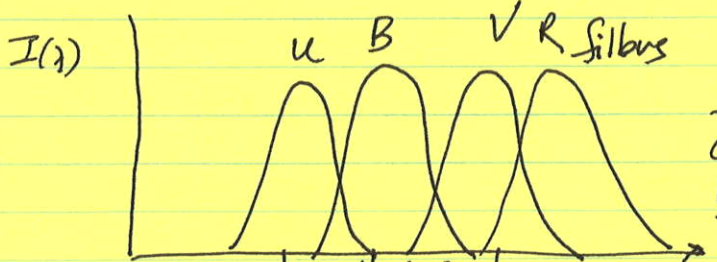
from the SB equation, define the effective temperature, T_{eff} . from,

$$L_* = 4\pi R_*^2 \sigma T_{\text{eff}}^4$$

$$\Rightarrow T_{\text{eff}}^4 = \frac{L_*}{4\pi\sigma R_*^2}$$

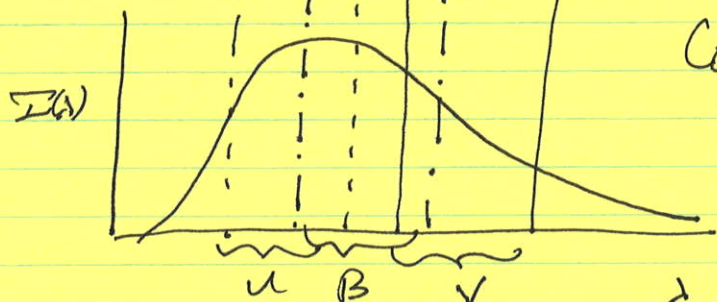
T_{eff} doesn't necessarily agree w/ T from Wien law
(*s aren't blackbodies)

Comment: Broad Band Photometry



Johnson Photometric system

	$\bar{\lambda}$	$\Delta\lambda$	T_m
u	365 nm	68 nm	~40% (??)
B	440 nm	98 nm	~40%
V	520 nm	90 nm	~65%
R	700 nm	220 nm	
I	900 nm	240 nm	



Compare energy in each band to get colors

Magnitude Scale

Pogson (1856) noted the brightest stars differed in brightness from the faintest stars by about a factor of 100.

The magnitude difference was 5 between the top and bottom of a logarithmic scale \Rightarrow

$$\Delta M = 1 \Rightarrow \frac{I_2}{I_1} = 10^{1/5}$$

or in general,

$$\frac{I_2}{I_1} = 10^{0.2(\Delta M)}$$

$$\Rightarrow \frac{2\Delta M}{5} = \log_{10} \left(\frac{I_2}{I_1} \right) = \log_{10} I_2 - \log_{10} I_1$$

$$\text{or } \Delta M = 2.5 \log_{10} \left(\frac{I_2}{I_1} \right)$$

defines apparent magnitude

I've observed the stars of 2 & 5

$$\Rightarrow \Delta M = 2.5 \left[\log_{10} \left(\frac{L_2}{4\pi d_2^2} \right) - \log_{10} \left(\frac{L_1}{4\pi d_1^2} \right) \right]$$

Since we let $L_1 = L_2 = L_*$ had circular $d_2 = 10 \text{ pc}$,
by definition

$$\Rightarrow m - M = 2.5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right)^2 = 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right)$$

Figure 17.7 Apparent Magnitude

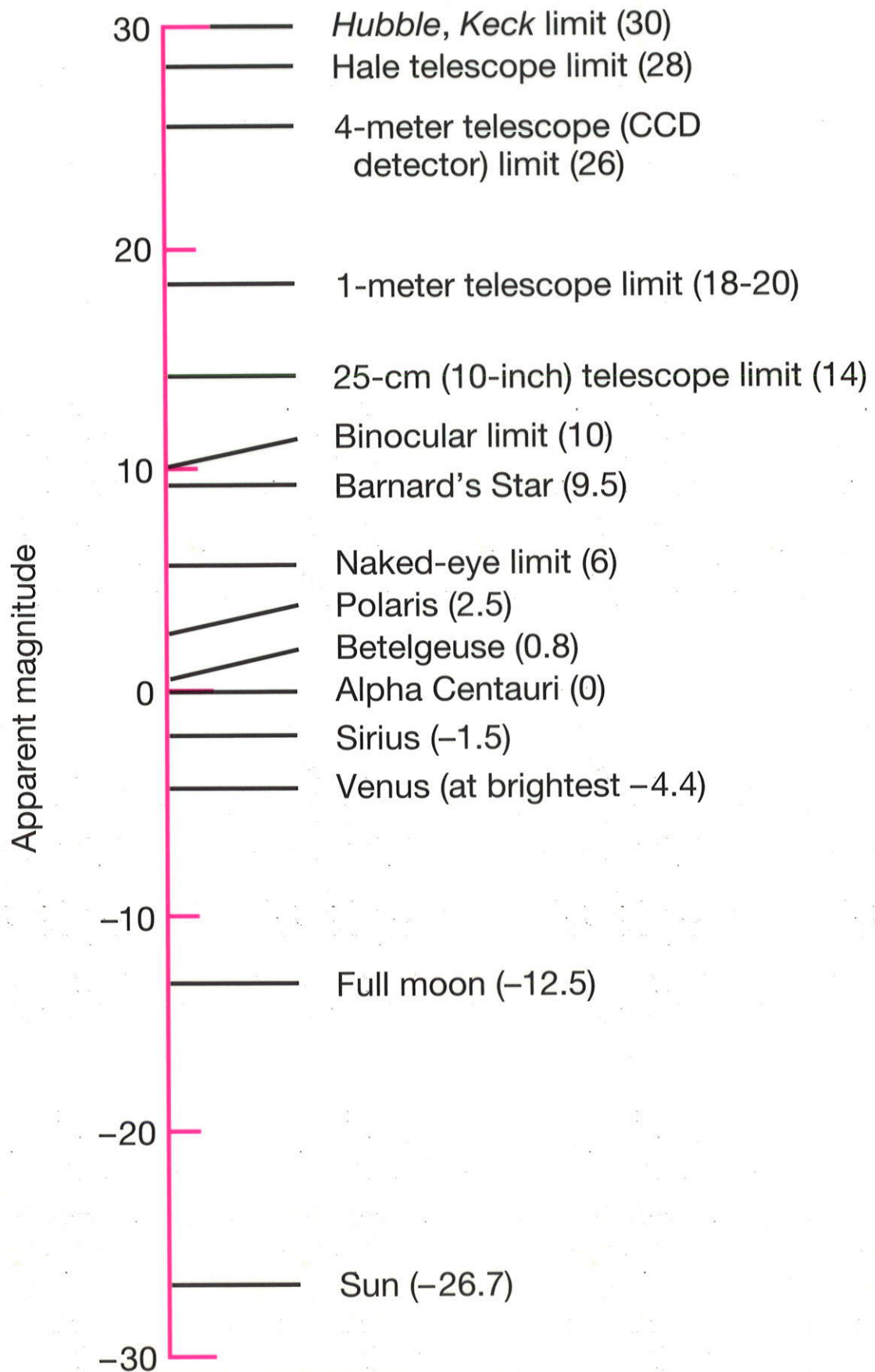


TABLE 17.1 Stellar Colors and Temperatures

B flux	Approximate Surface Temperature (K)	Color	Familiar Examples
1.3	30,000	blue-violet	Mintaka (δ Orionis)
1.2	20,000	blue	Rigel
1.00	10,000	white	Vega, Sirius
0.72	7000	yellow-white	Canopus
0.55	6000	yellow	Sun, Alpha Centauri
0.33	4000	orange	Arcturus, Aldebaran
0.21	3000	red	Betelgeuse, Barnard's Star

⑬ line spectroscopy (from spectra)



a) $E = T + W$ for virial theorem, $2T + W = 0$

$$\rightarrow \boxed{E = \frac{1}{2}W + W = +\frac{1}{2}W = -\frac{e^2}{2r}}$$

b) $-\frac{e^2}{r^2} + \frac{mv^2}{r} = 0$, force balance

quantized angular momentum \rightarrow in QM, $mvr = n\hbar$, $v = \frac{n\hbar}{mr}$

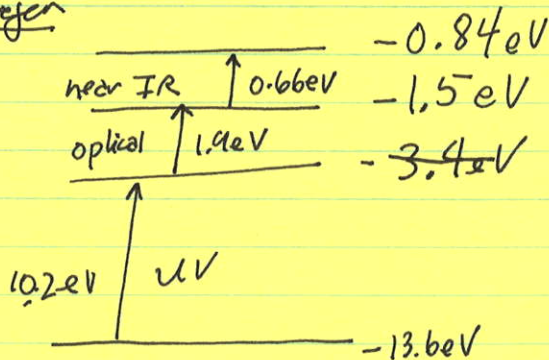
$$\Rightarrow -\frac{e^2}{r^2} + \frac{m}{r} \left(\frac{n^2 \hbar^2}{m^2 r^2} \right) = 0$$

$$\Rightarrow \boxed{r_n = \frac{n^2 \hbar^2}{me^2} = \text{Bohr radius}} \quad \text{Bohr radius}$$

$$\Rightarrow \boxed{E_n = -\frac{e^2}{2} \left(\frac{me^2}{n^2 \hbar^2} \right)}$$

$$= -\frac{13.6 \text{ eV}}{n^2}$$

hydrogen



note: $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

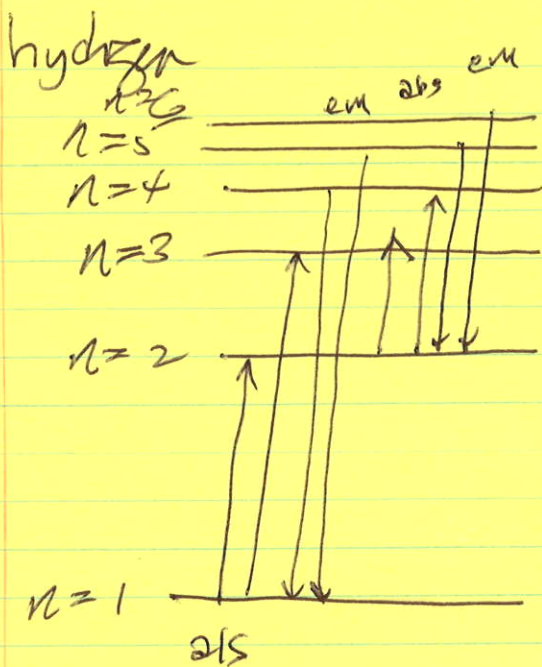
$$\Rightarrow 10 \text{ eV} = 1.602 \times 10^{-18} \text{ J}$$

$$\& \frac{hc}{\lambda} = 1.60 \times 10^{-18} \text{ J}$$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.60 \times 10^{-18}} \text{ m}$$

$$\approx 1.3 \times 10^{-7} \text{ m}$$

$$\approx 1300 \text{ \AA} \text{ in near UV}$$



① ends or starts at $n=1$

uv } Lyman series, $\Delta n = 1$
 $= 2 \beta$
 $= 3 \gamma$
 $= 4 \delta$
 $= 5 \epsilon$

② ends or starts at $n=2$

opt } Balmer series

③ ends or starts at $n=3$

opt, IR } Paschen series

④ ends or starts at $n=4$

IR } Brackett series

⑤ ends or starts at $n=5$

IR } Pfund series

e.g. \rightarrow Lyman α , $-13.6 \text{ eV} \rightarrow -3.4 \text{ eV} = 10.2 \text{ eV}$

$$E_{\text{ph}} = 10.2 \text{ eV} \times \frac{1.602 \times 10^{-19} \text{ eV}}{1 \text{ eV}}$$

$$= 1.63 \times 10^{-18} \text{ eV}$$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = 1.216 \times 10^{-5} \text{ cm}$$

$$= 1,216 \text{ \AA}$$

$$1 \text{ \AA} = 10^{-8} \text{ cm}$$

Excitation

$$a) E = 1.63 \times 10^{-11} \text{ eV}$$

$$(i) E = kT_{ex} = 1.63 \times 10^{-11} \text{ eV} = 1.38 \times 10^{-16} T_{ex}$$

$$\Rightarrow T_{ex} \approx 1.2 \times 10^5 \text{ K}$$

(ii) actually it's the tail that excites \Rightarrow
 $T < T_{ex}$

$$b) E \approx 13.6 \text{ eV} \Rightarrow T_{ion} \approx 2 \times 10^5 \text{ K}$$

(ii) actually tail excites $\Rightarrow T_{ion} < T_{ion}$