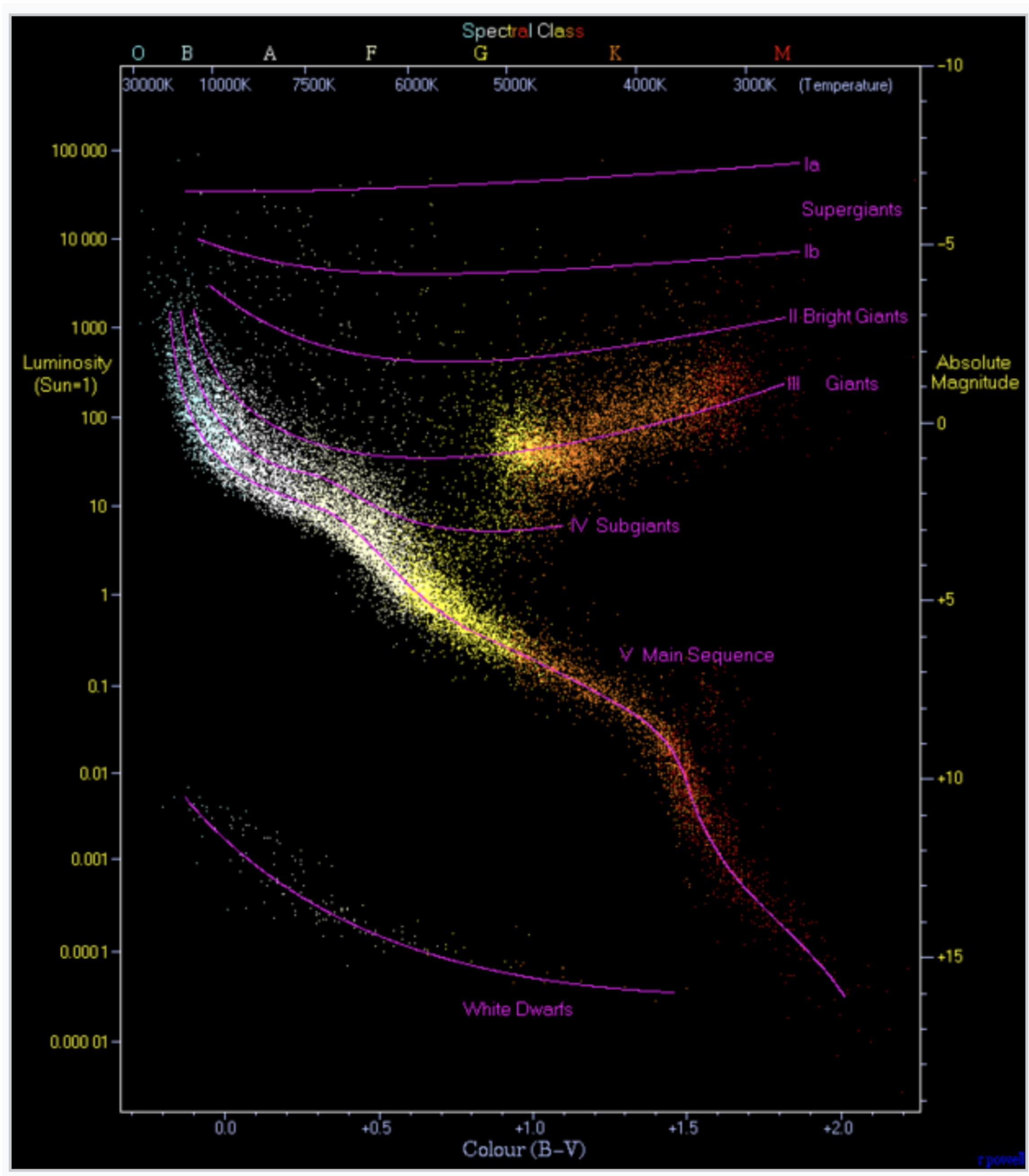
NAME ____

Astronomy 321
Test 1
January 30, 2023

Question 1:

A Hertzsprung-Russell (HR) diagram constructed from Hipparcos data is shown below. The diagram shows the luminosity of a star L_* and its absolute magnitude plotted against its temperature T, spectral class, and color index.



a. The bulk of the observable stars in our Galaxy are found on the Main Sequence. Roughly, what fraction of observable stars is found along the Main Sequence and what is the significance of this result in terms of the evolution of a star? What are the longest lived phases of a star's life according to the HR diagram?

b. Based on the observed properties of stars, it appears that stars do not evolve greatly on the Main Sequence, they do not start out as O stars and then cool changing into M stars over their lifetimes as once thought. However, draw a line of constant radius on the HR diagram. For definiteness, draw the line for constant $R_\odot$, the radius of the Sun. What do you make of this line? Why are we certain that this line does not show that stars evolve along the Main Sequence? Cite data that shows stars do not evolve greatly while on the Main Sequence.

c. The Main Sequence is not a line in the HR diagram, it shows a spread on the diagram. The spread is not due to observational error, it likely has physical origins. Suppose the spread is caused by evolution of stars while they are on the Main Sequence. If the Sun evolves on the Main Sequence with constant luminosity, L, how much must the temperature of the Sun change over its Main Sequence lifetime to explain the spread in the temperature of the Main Sequence at the location of the Sun?

d. Imagine that the Sun evolves on the Main Sequence at constant temperature, T. In this case, how much must the luminosity of the Sun change over its Main Sequence lifetime to explain the spread in luminosity of the Main Sequence at the Sun's location?

   Over its Main Sequence lifetime, both the luminosity and temperature of the Sun do in fact change, L goes from 0.7 to 1.84 $L_\odot$ and T_{eff} goes from 5596 K to 5820 K, and back to 5751 K.

Question 2:

Verify that Kepler's third law can be written in the form

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3 \tag{1}$$

where P is the orbital period, G is the gravitational constant, m_1 and m_2 are the masses of the objects, and $a = a_1 + a_2$ is the sum of the semi-major axes. For simplicity consider circular orbits. Show that the modified version of Kepler's Third Law applies to the four moons of Jupiter discovered by Galileo (the Galilean moons: Io, Europa, Ganymede, and Europa).

a. Using the data available in Appendix: Solar System Data, create a plot of $\log_{10} P$ vs $\log_{10} a$.

Appendix: Solar System Data

Data of Selected Major Satellites

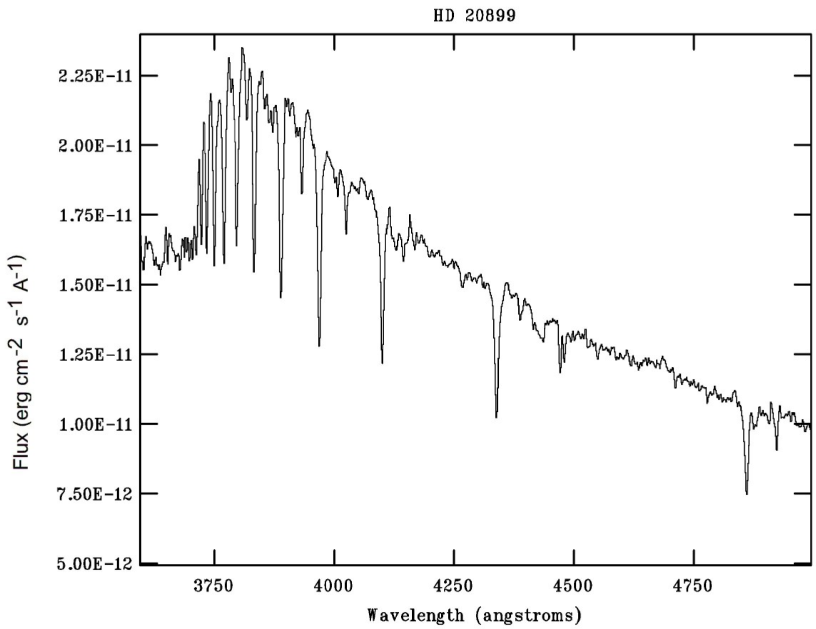
Satellite	Parent Planet	Mass (10^{22} kg)	Radius (10^3 km)	Density (kg m^{-3})	Orbital Period (d)	Semimajor Axis (10^3 km)
Moon	Earth	7.349	1.7371	3350	27.322	384.4
Io	Jupiter	8.932	1.8216	3530	1.769	421.6
Europa	Jupiter	4.800	1.5608	3010	3.551	670.9
Ganymede	Jupiter	14.819	2.6312	1940	7.155	1070.4
Callisto	Jupiter	10.759	2.4103	1830	16.689	1882.7
Titan	Saturn	13.455	2.575	1881	15.945	1221.8
Triton	Neptune	2.14	1.3534	2050	5.877	354.8

b. From the graph, show that the slope of the best-fit straight line through the data is 3/2.

c. Calculate the mass of Jupiter from the value of the y-intercept.

This is problem 2.12 from the text.

Question 3:



Jacoby, G. H., Hunter, D. A., and Christian, C. A. 1984, Astrophys. J. Supp. Ser., 56, 257

a. Estimate the surface temperature for the above star using the methods of the homework assignment. How do your estimates for the T of the star compare? If the temperatures differ, explain why they differ.

b. Estimate the temperature of the star based on the slope of the spectrum for the region $hc/\lambda < kT$. Use the full Planck spectrum for this estimate..

c. What is the approximate spectral class for the star?

d. Identify the hydrogen lines and their wavelengths in the star's spectrum. Where does the Balmer edge fall, the wavelengths for which photons with shorter wavelengths can ionize a hydrogen atom when the electron is in its first excited state.

Question 4

The following Table contains data for Main Sequence stars. Except for the Spectral classes and T_{eff}, which is in Kelvin, all quantities are in Solar units ($M_\odot$, $L_\odot$, and $R_\odot$).

a. Find scaling laws for $R(M_*)$, $L(M_*)$, and $T_{eff}(M_*)$. You may use log-log graph paper rather than calculating logs.

b. Write the relations for $L_*(T_{eff})$, $R_*(T_{eff})$, and $M_*(T_{eff})$.

c. How closely can the relationships be described as power laws?

d. Are there changes in slope of the scaling relations as functions of the stellar mass M_*? If so what might they mean? (That is, do *kinks* in plots for the various stellar properties as functions of M_* have physical meaning?)

Main-Sequence Stars (Luminosity Class V)

Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
O5	42000	499000	13.4	60	−9.51	−4.40	−5.1	−1.19	−0.33
O6	39500	324000	12.2	37	−9.04	−3.93	−5.1	−1.17	−0.33
O7	37500	216000	11.0	—	−8.60	−3.68	−4.9	−1.15	−0.32
O8	35800	147000	10.0	23	−8.18	−3.54	−4.6	−1.14	−0.32
B0	30000	32500	6.7	17.5	−6.54	−3.16	−3.4	−1.08	−0.30
B1	25400	9950	5.2	—	−5.26	−2.70	−2.6	−0.95	−0.26
B2	20900	2920	4.1	—	−3.92	−2.35	−1.6	−0.84	−0.24
B3	18800	1580	3.8	7.6	−3.26	−1.94	−1.3	−0.71	−0.20
B5	15200	480	3.2	5.9	−1.96	−1.46	−0.5	−0.58	−0.17
B6	13700	272	2.9	—	−1.35	−1.21	−0.1	−0.50	−0.15
B7	12500	160	2.7	—	−0.77	−1.02	+0.3	−0.43	−0.13
B8	11400	96.7	2.5	3.8	−0.22	−0.80	+0.6	−0.34	−0.11
B9	10500	60.7	2.3	—	+0.28	−0.51	+0.8	−0.20	−0.07
A0	9800	39.4	2.2	2.9	+0.75	−0.30	+1.1	−0.02	−0.02
A1	9400	30.3	2.1	—	+1.04	−0.23	+1.3	+0.02	+0.01
A2	9020	23.6	2.0	—	+1.31	−0.20	+1.5	+0.05	+0.05
A5	8190	12.3	1.8	2.0	+2.02	−0.15	+2.2	+0.10	+0.15
A8	7600	7.13	1.5	—	+2.61	−0.10	+2.7	+0.09	+0.25
F0	7300	5.21	1.4	1.6	+2.95	−0.09	+3.0	+0.03	+0.30
F2	7050	3.89	1.3	—	+3.27	−0.11	+3.4	+0.00	+0.35
F5	6650	2.56	1.2	1.4	+3.72	−0.14	+3.9	−0.02	+0.44
F8	6250	1.68	1.1	—	+4.18	−0.16	+4.3	+0.02	+0.52

Appendix: Stellar Data

Main-Sequence Stars (Luminosity Class V)

Sp. Type	T_e (K)	$L/L_\odot$	$R/R_\odot$	$M/M_\odot$	M_{bol}	BC	M_V	$U-B$	$B-V$
G0	5940	1.25	1.06	1.05	+4.50	−0.18	+4.7	+0.06	+0.58
G2	5790	1.07	1.03	—	+4.66	−0.20	+4.9	+0.12	+0.63
Sun[a]	5777	1.00	1.00	1.00	+4.74	−0.08	+4.82	+0.195	+0.650
G8	5310	0.656	0.96	—	+5.20	−0.40	+5.6	+0.30	+0.74
K0	5150	0.552	0.93	0.79	+5.39	−0.31	+5.7	+0.45	+0.81
K1	4990	0.461	0.91	—	+5.58	−0.37	+6.0	+0.54	+0.86
K3	4690	0.318	0.86	—	+5.98	−0.50	+6.5	+0.80	+0.96
K4	4540	0.263	0.83	—	+6.19	−0.55	+6.7	—	+1.05
K5	4410	0.216	0.80	0.67	+6.40	−0.72	+7.1	+0.98	+1.15
K7	4150	0.145	0.74	—	+6.84	−1.01	+7.8	+1.21	+1.33
M0	3840	0.077	0.63	0.51	+7.52	−1.38	+8.9	+1.22	+1.40
M1	3660	0.050	0.56	—	+7.99	−1.62	+9.6	+1.21	+1.46
M2	3520	0.032	0.48	0.40	+8.47	−1.89	+10.4	+1.18	+1.49
M3	3400	0.020	0.41	—	+8.97	−2.15	+11.1	+1.16	+1.51
M4	3290	0.013	0.35	—	+9.49	−2.38	+11.9	+1.15	+1.54
M5	3170	0.0076	0.29	0.21	+10.1	−2.73	+12.8	+1.24	+1.64
M6	3030	0.0044	0.24	—	+10.6	−3.21	+13.8	+1.32	+1.73
M7	2860	0.0025	0.20	—	+11.3	−3.46	+14.7	+1.40	+1.80

[a]Values adopted in this text.