



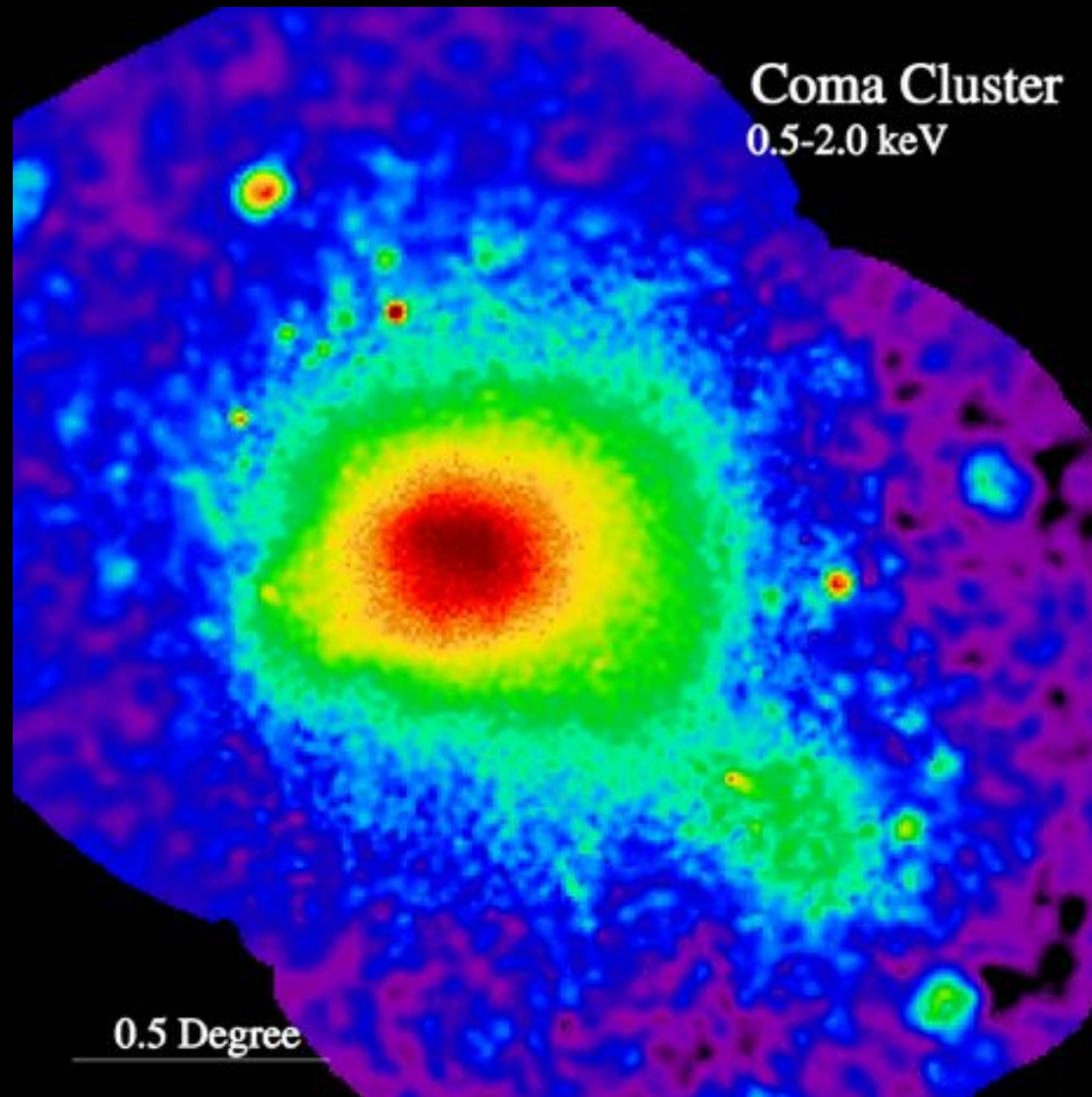
## DARK MATTER

Around 30 % of the Universe is matter of which  
~26-27 % is dark matter.

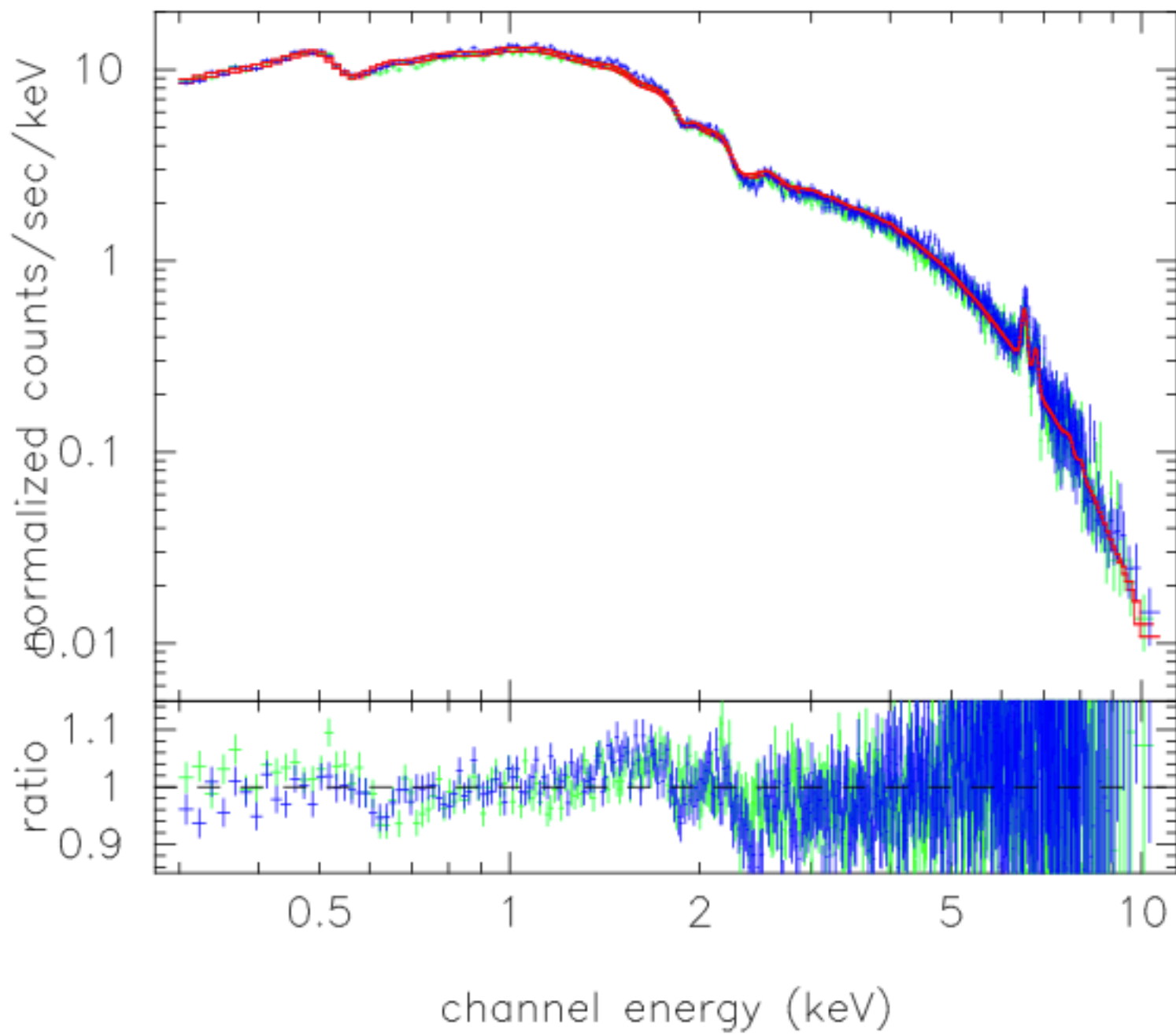


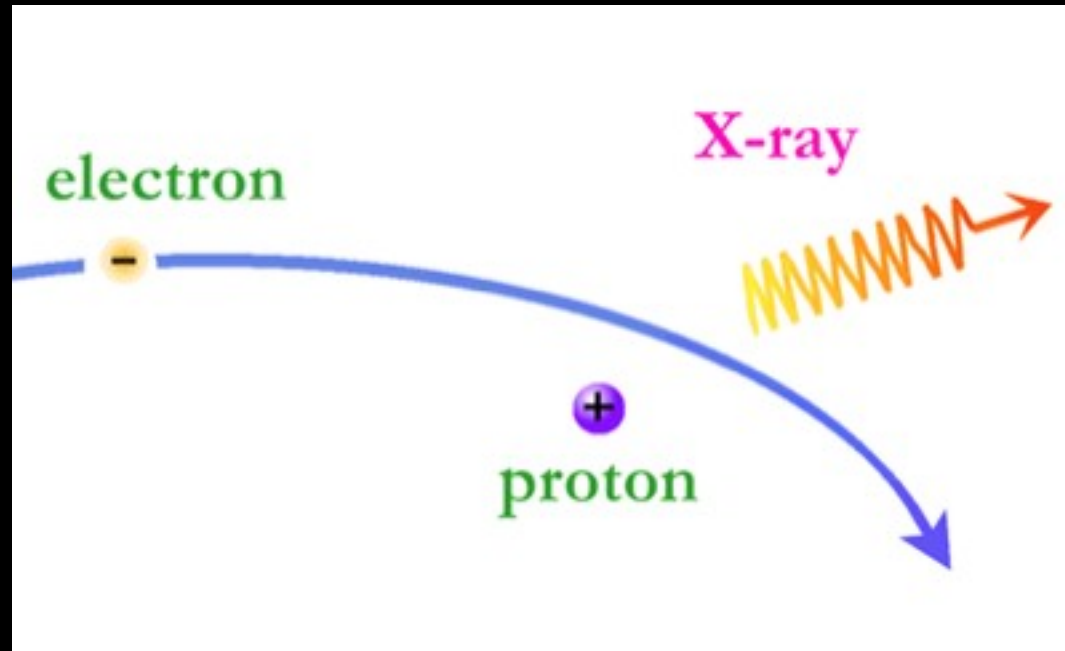
## COMA Cluster

1.  $>10,000$  members,  $R \sim 3$  Mpc,  $M \sim 7 \times 10^{14} M_{\odot}$
2.  $D \sim 99$  Mpc,  $z = 0.231$
3.  $\sigma \sim 900$  km/s



COMA Cluster  
Soft x-ray image obtained by ROSAT

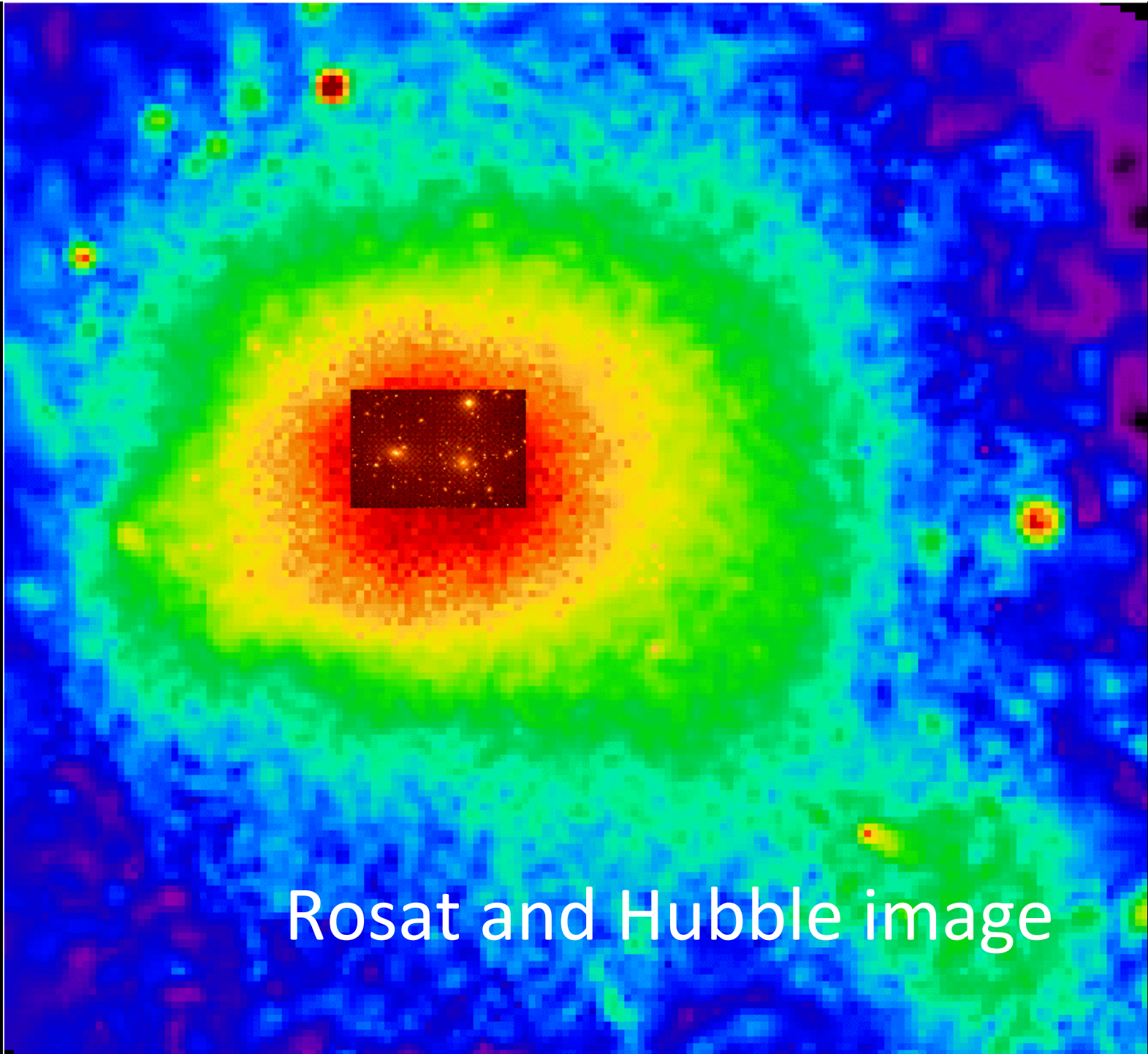




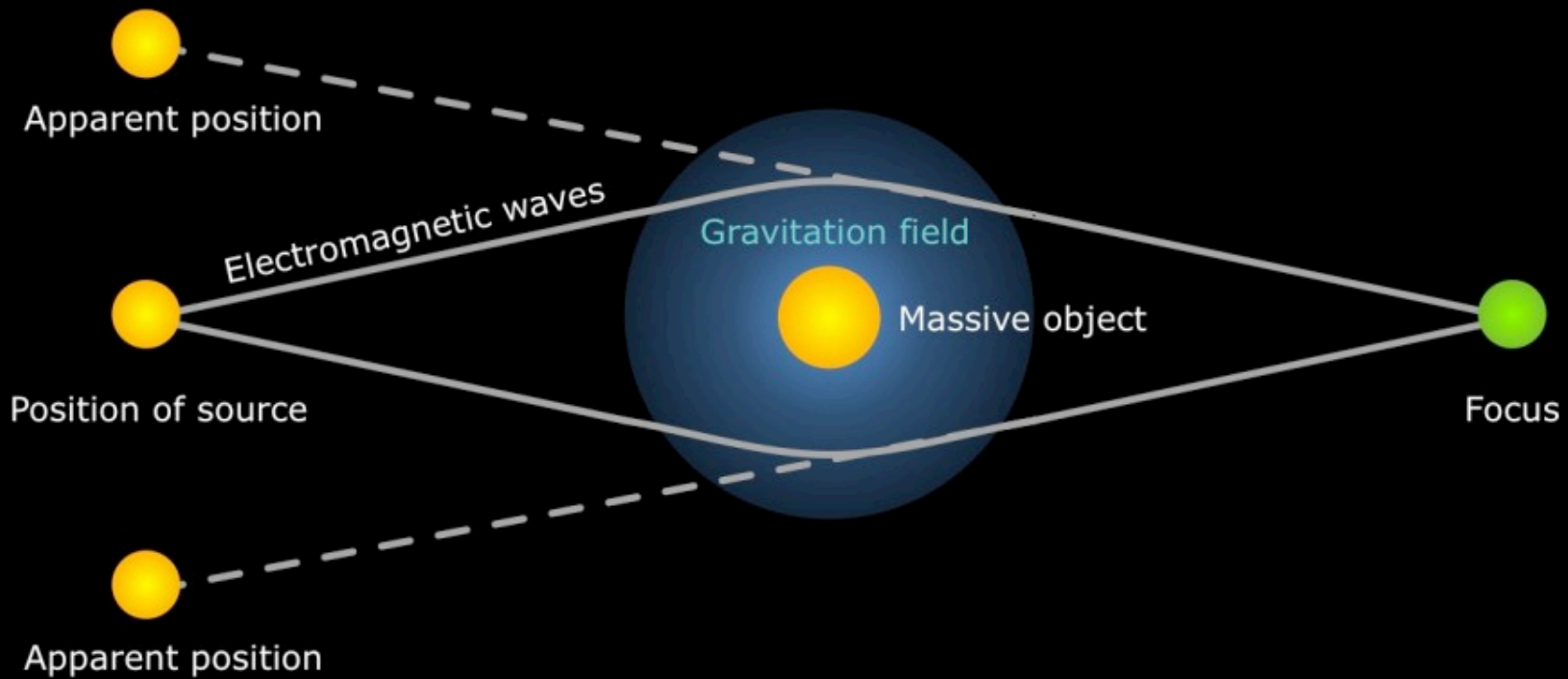
## Bremsstrahlung (free-free) Emission

$$\epsilon_\nu = 6.8 \times 10^{-38} n_e n_i Z^2 g_{\text{ff}} T^{-1/2} e^{-h\nu/kT} \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}$$

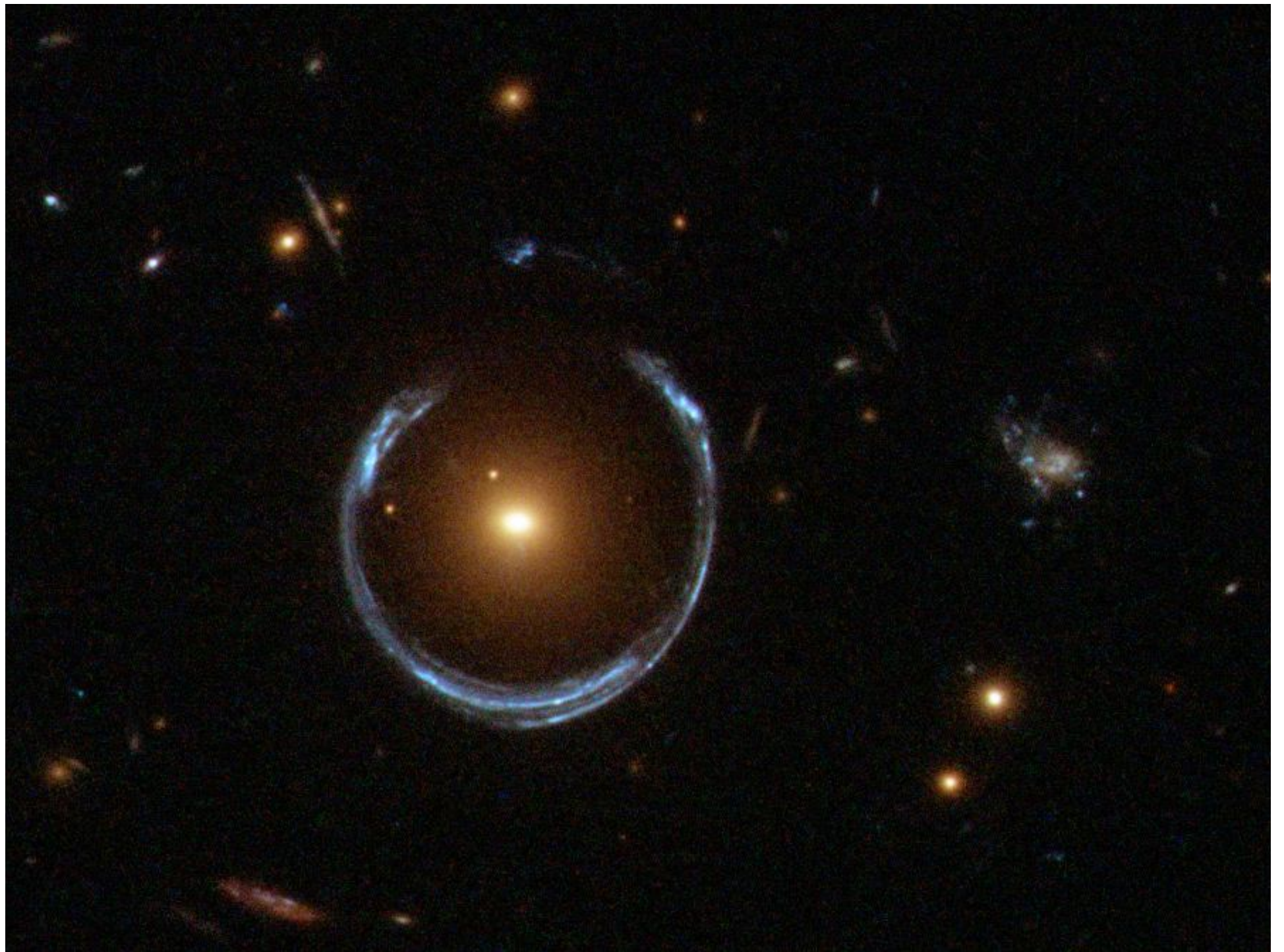
$$\epsilon = 2.4 \times 10^{-27} n_e n_i Z^2 g_{\text{ff}} T^{1/2} \text{ erg s}^{-1} \text{ cm}^{-3}$$



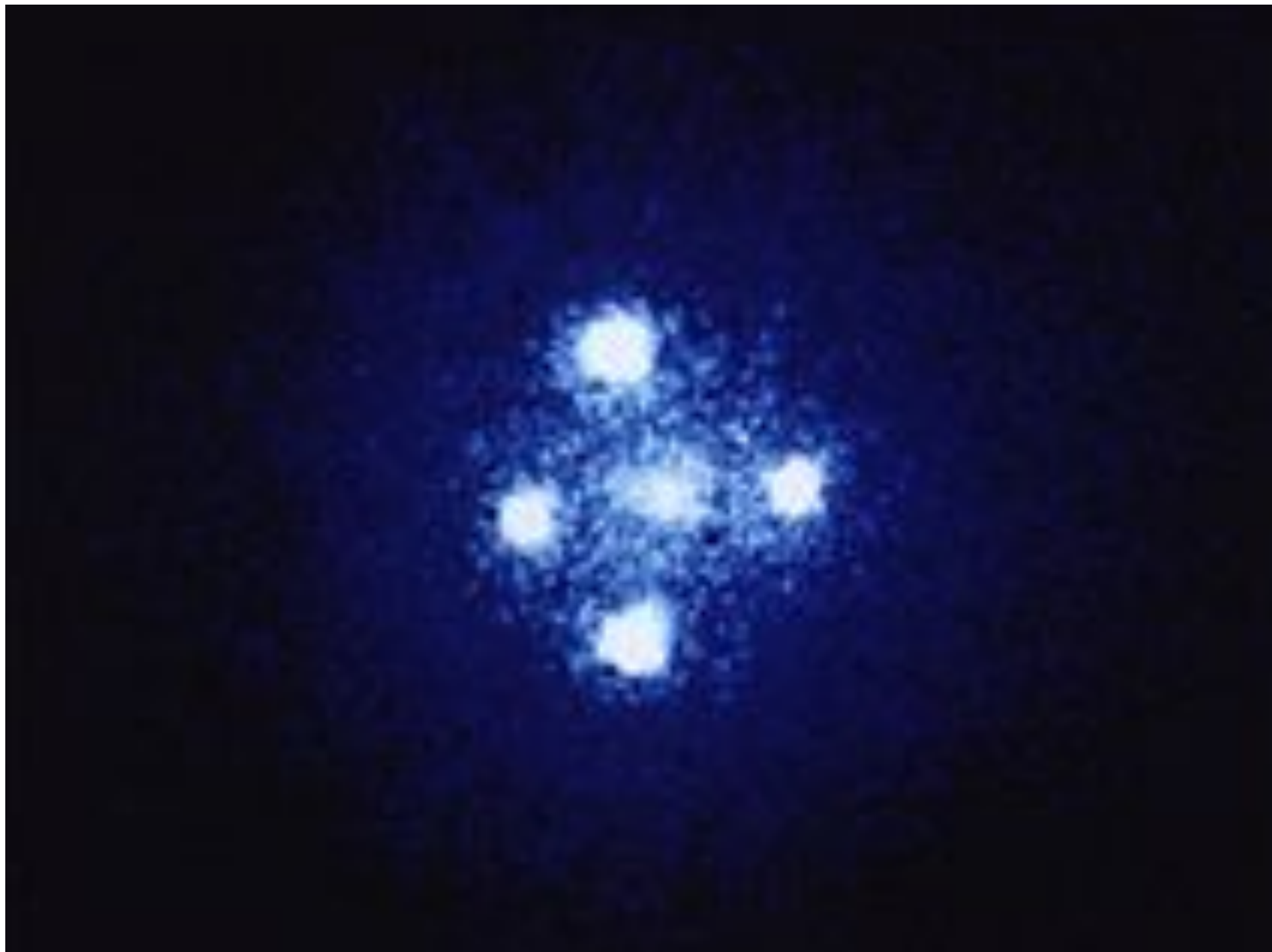
Rosat and Hubble image



# Schematic of Gravitational Lensing









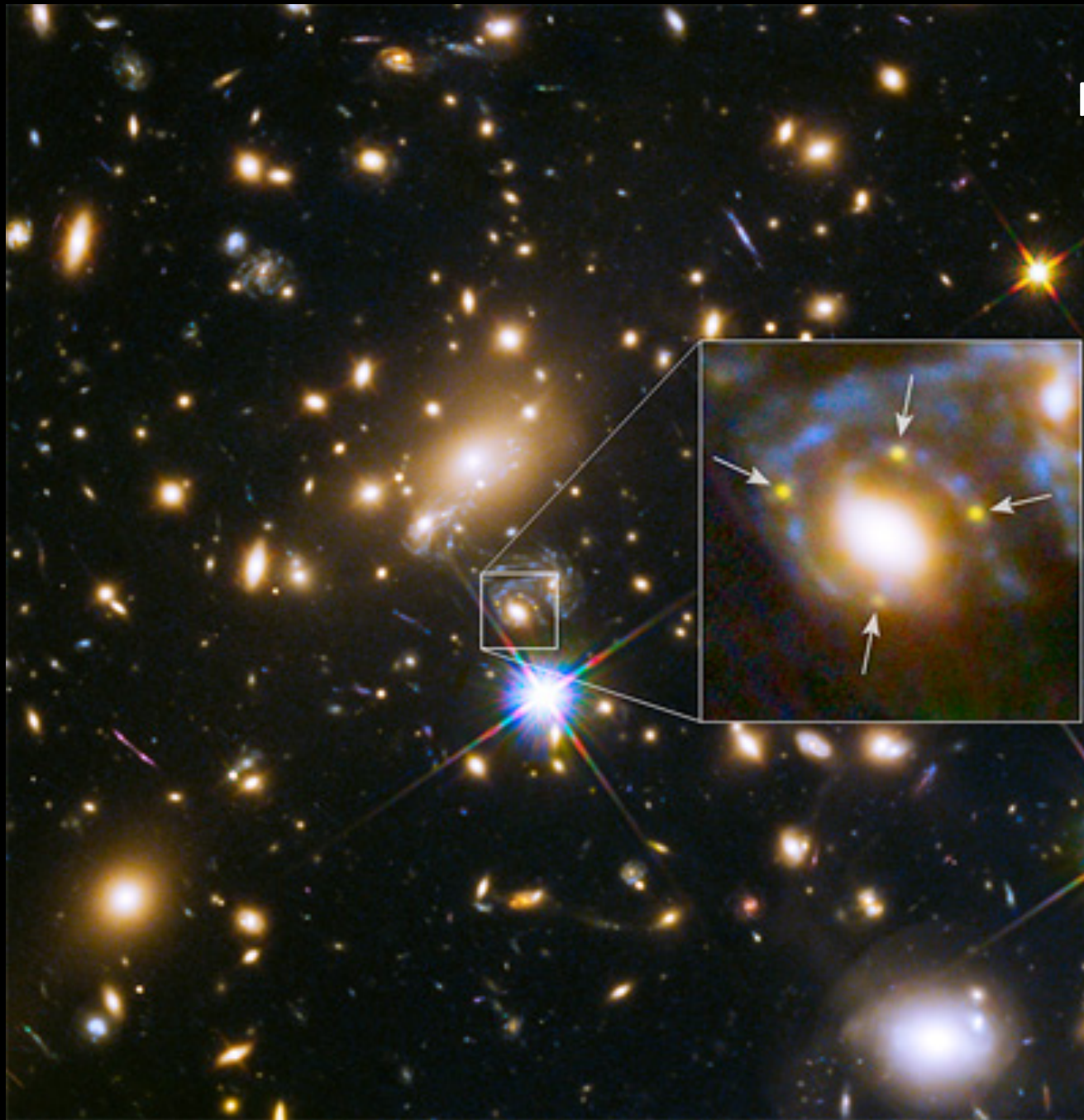


**Distant Galaxy Lensed by Cluster Abell 2218**  
**Hubble Space Telescope • WFPC2 • ACS**

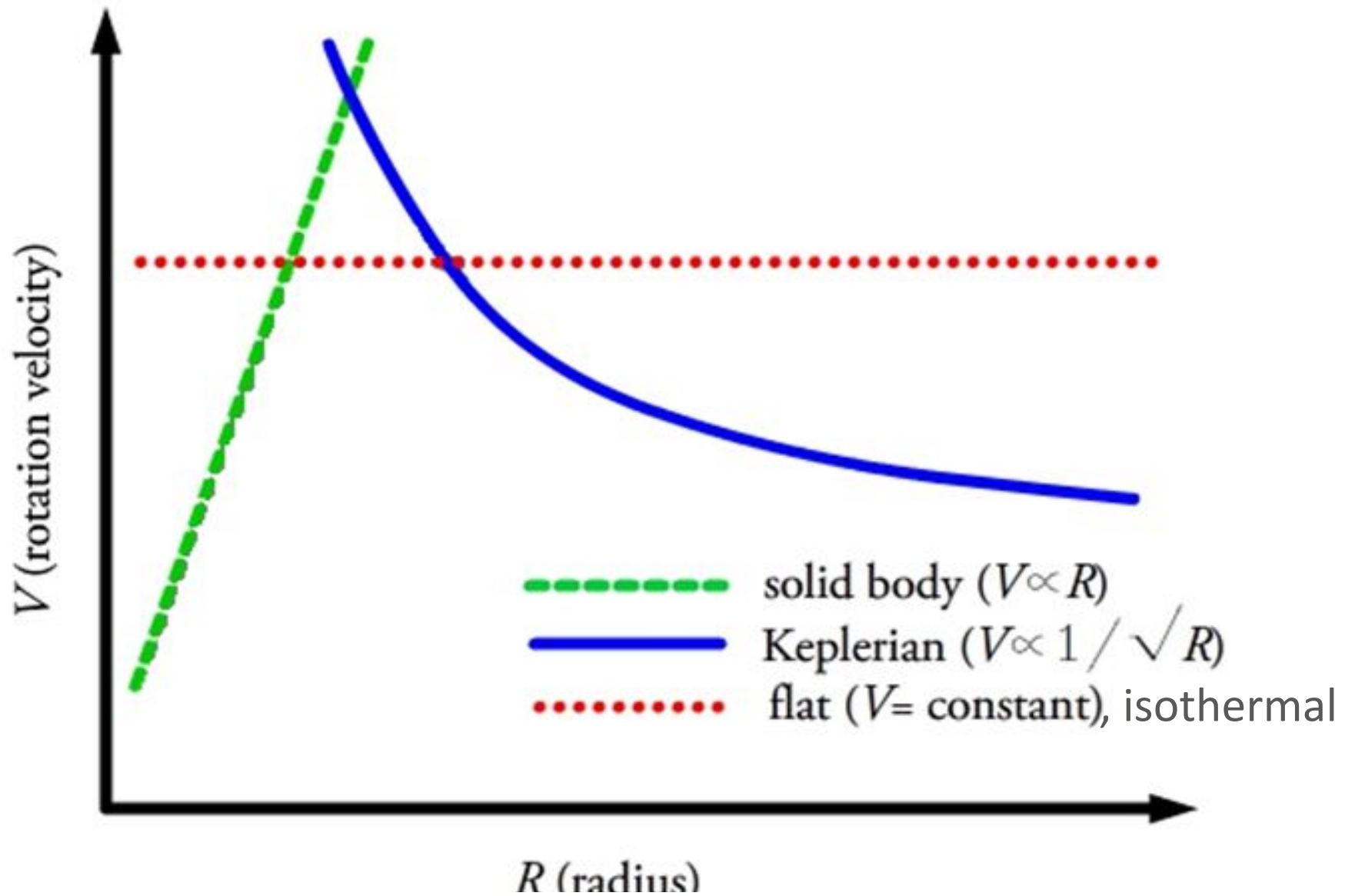
ESA, NASA, J.-P. Kneib (Caltech/Observatoire Midi-Pyrénées) and R. Ellis (Caltech)

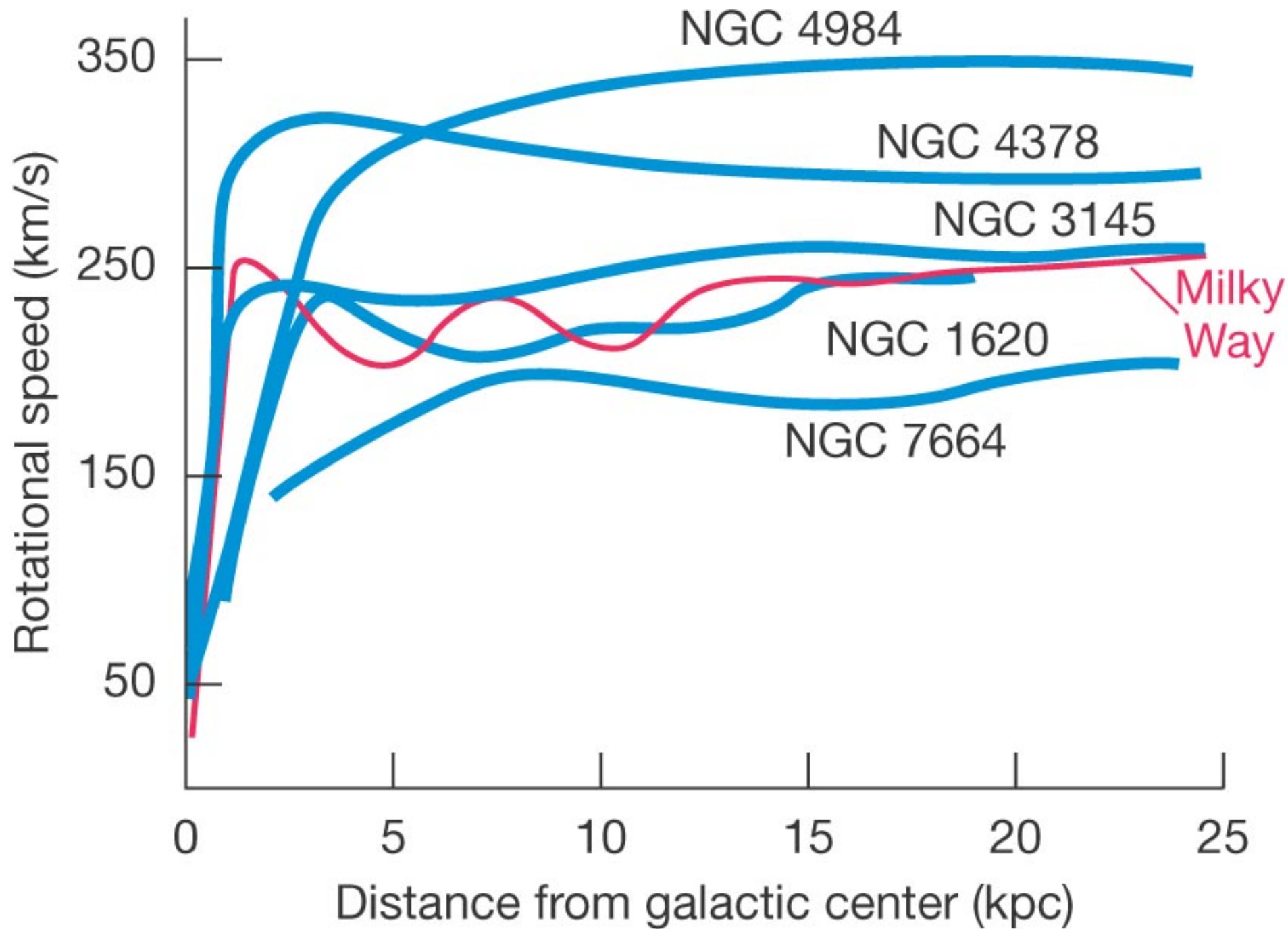
STScI-PRC04-08

MACS J1149.6+2223  
lenses background  
Supernova

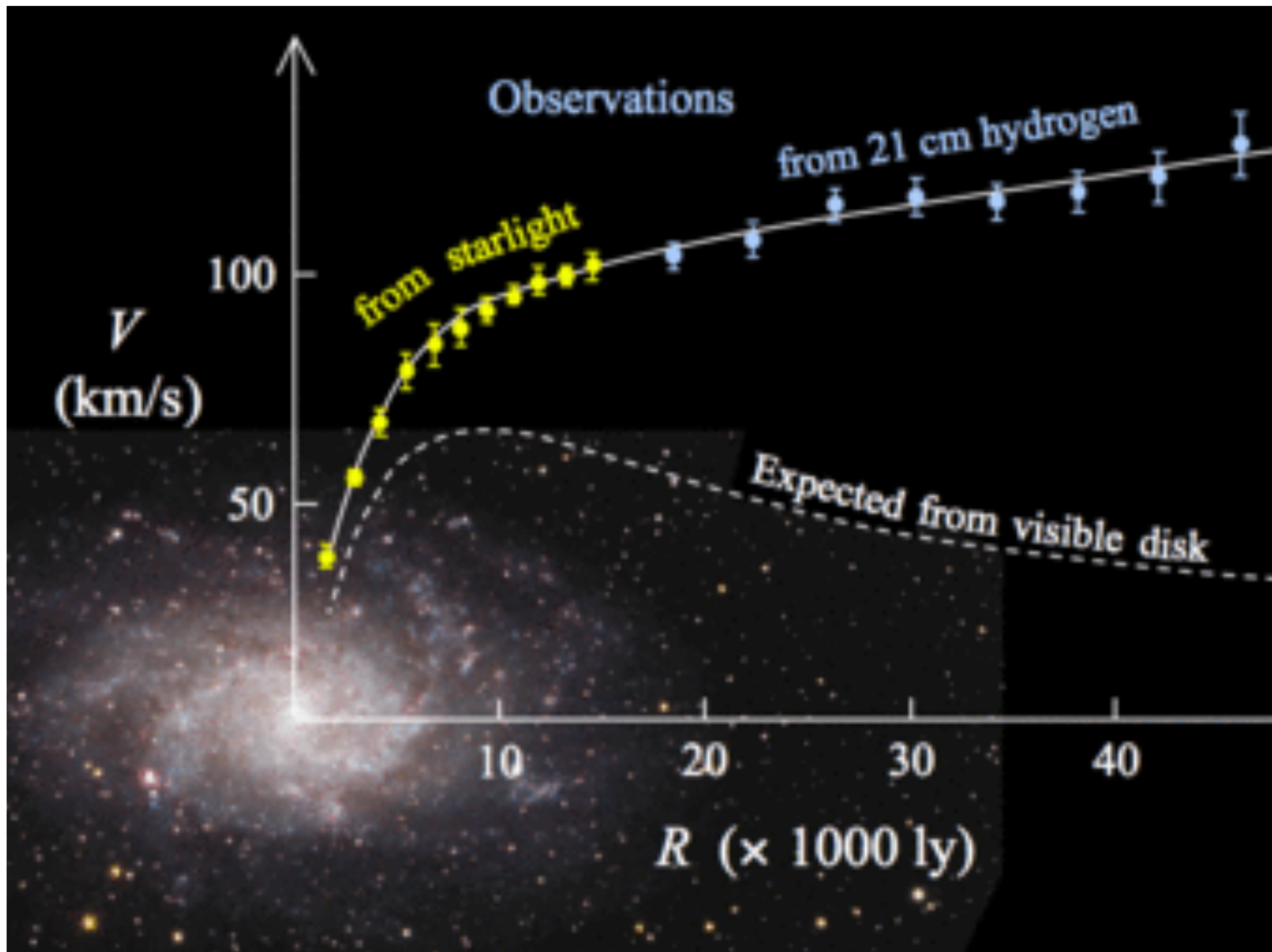


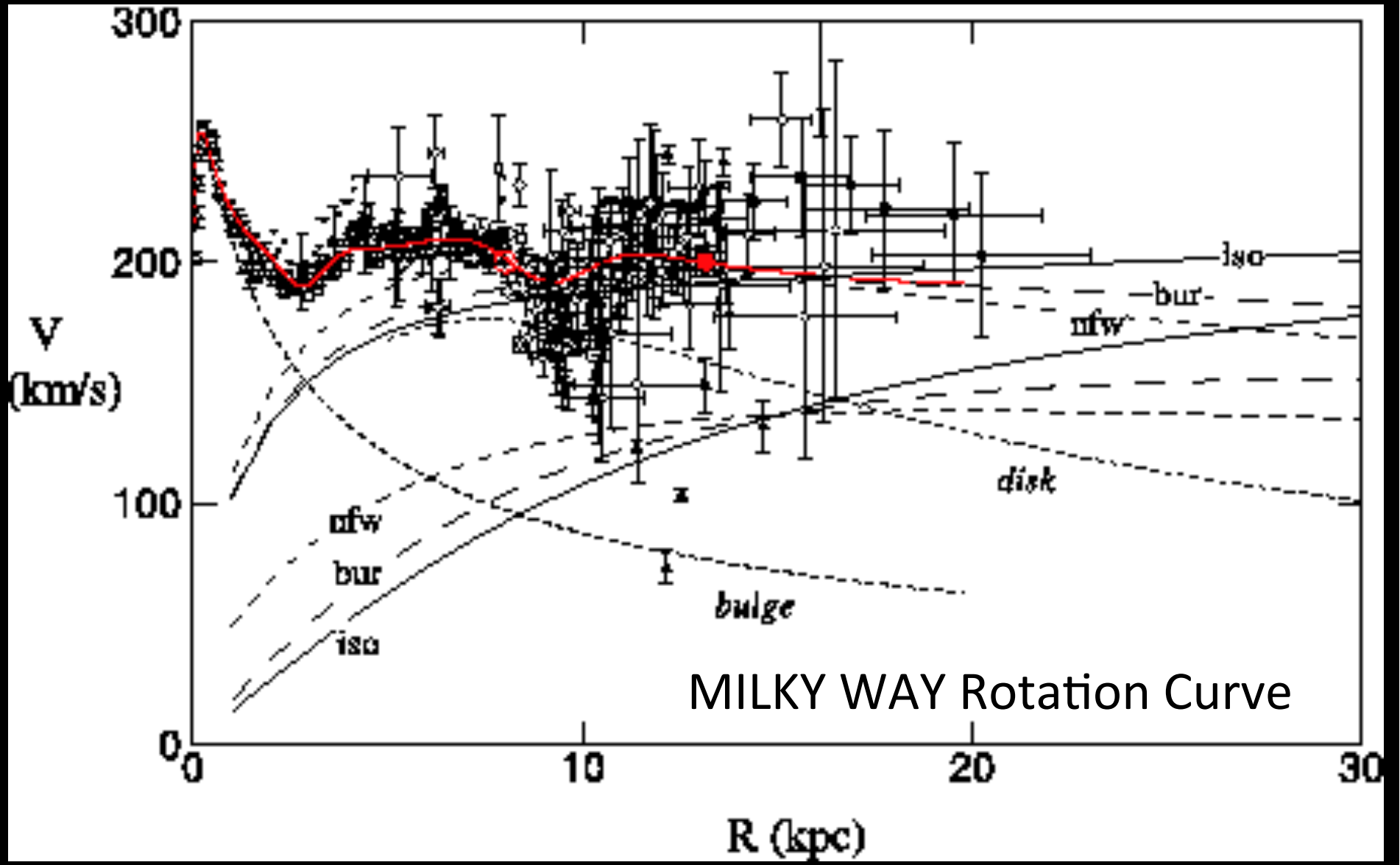
# Rotation Curves





(b)





MILKY WAY Rotation Curve





Passage of a MACHO in front of a distant star

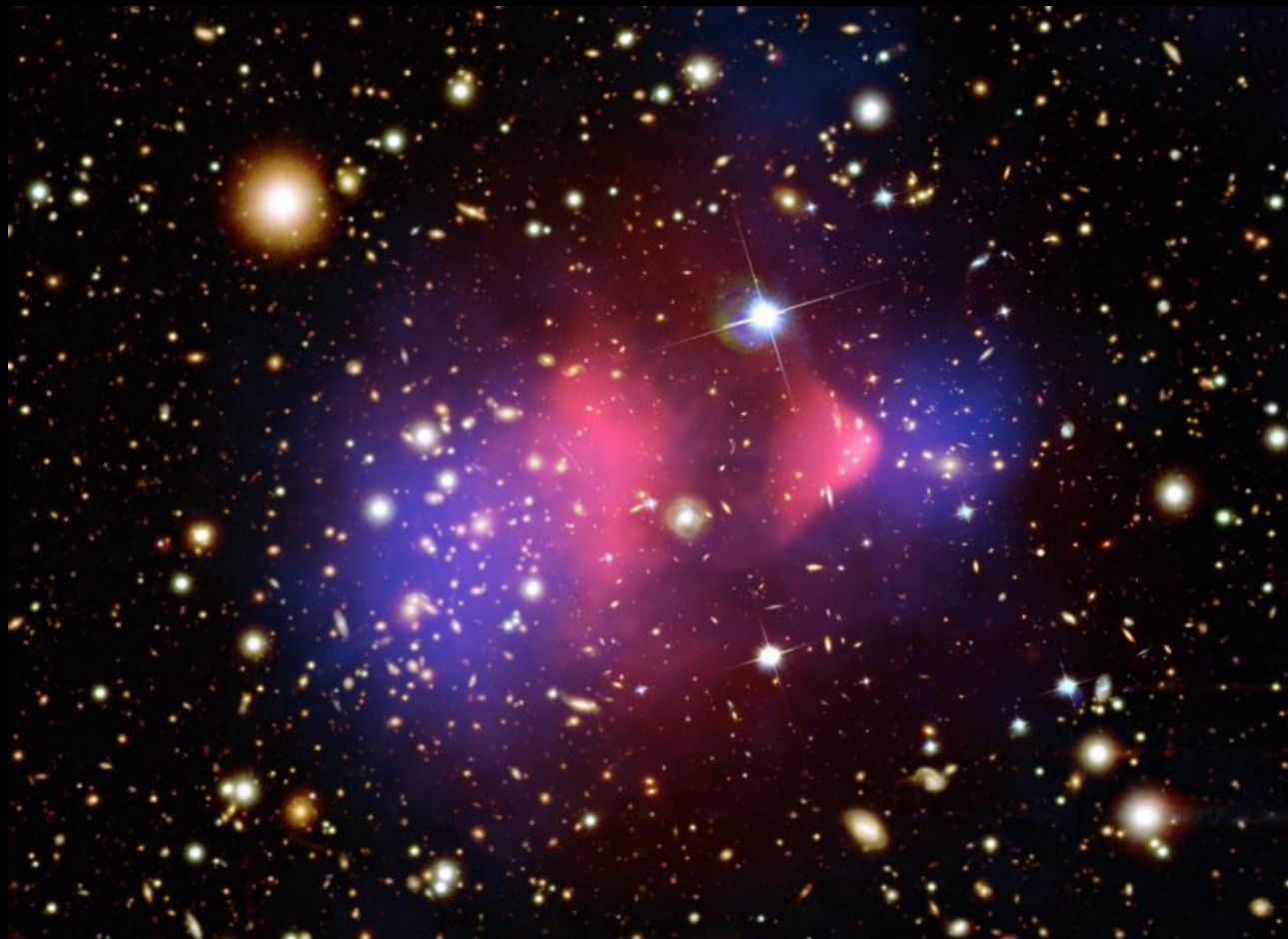
## MACHO Results

Analysis of 5.7 years of photometry on 11.9 million stars in the LMC reveals 13-17 microlensing events. A detailed treatment of our detection efficiency shows that this is significantly more than the 2 to 4 events expected from lensing by known stellar populations. The timescales of the events range from 34 to 230 days. We estimate the microlensing optical depth towards the LMC from events with  $2 < t < 400$  days to be  $1.2 \times 10^{-7}$  with an additional 20%-30% systematic error. The spatial distribution of events is mildly inconsistent with LMC/LMC disk self-lensing, but is consistent with an extended lens distribution such as a Milky Way or LMC halo. Interpreted in the context of a Galactic dark matter halo, consisting partially of compact objects, a maximum likelihood analysis gives MACHO halo fraction of 20% for a typical halo model with a 95% confidence interval of 8% to 50%. A 100% MACHO halo is ruled out at the 95% C.L. for all except our most extreme halo model. Interpreted as a Galactic halo population, the most likely MACHO mass is between 0.15 and 0.9 Solar masses, depending on the halo model, and the total mass in MACHOs out to 50 kpc is found to be  $9 \times 10^{10}$ , independent of the halo model.

## EROS and OGLE Re-Analysis

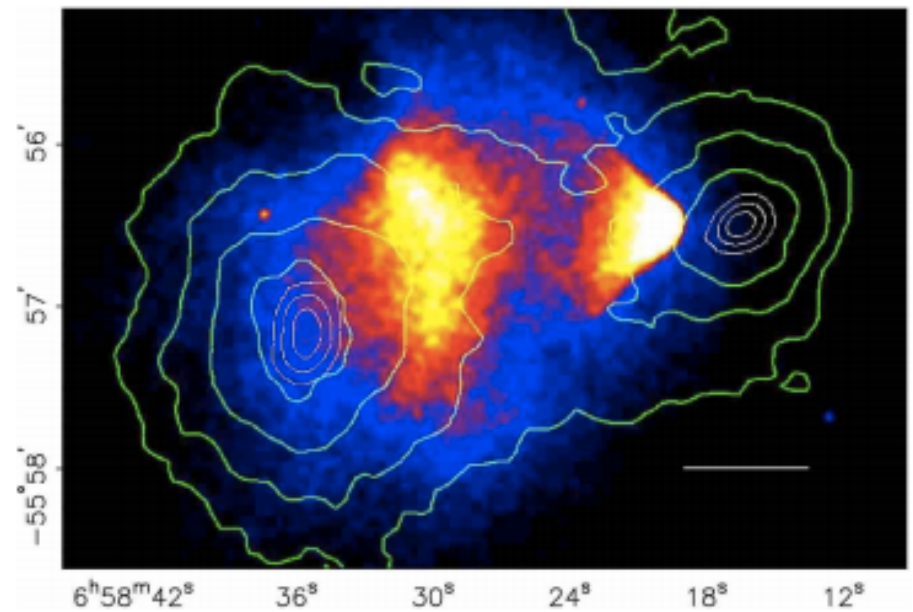
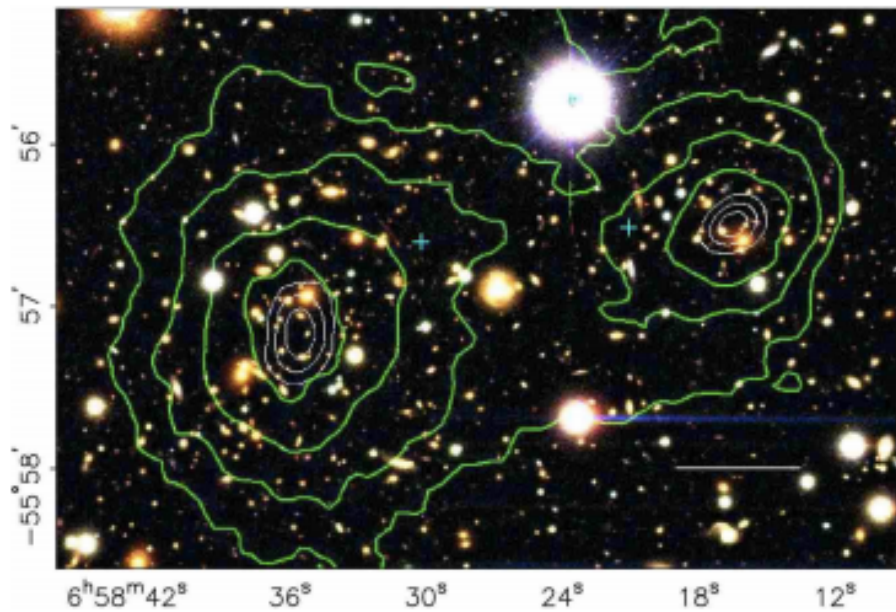
A new analysis of the results of the EROS-2, OGLE-II, and OGLE-III microlensing campaigns towards the Small Magellanic Cloud (SMC) shed light on the issue of the nature of the reported microlensing candidate events, whether to be attributed to lenses belonging to known population (the SMC luminous components or the Milky Way disc, to which we broadly refer to as "self lensing") or to the would be population of dark matter compact halo objects (MACHOs). Overall, of five reported microlensing events towards the SMC (one by EROS and four by OGLE), the re-analysis shows that in terms of number of events the expected self lensing signal may indeed explain the observed rate. However, the characteristics of the events, spatial distribution and duration (and for one event, the projected velocity) rather suggest a non-self lensing origin for a few of them. In particular the upper limit for the halo mass fraction in form of MACHOs given the expected self-lensing and MACHO lensing signal. At 95% CL, the tighter upper limit, about 10%, is found for MACHO mass of  $10-2M_{\odot}$ , upper limit that reduces to above 20% for  $0.5M_{\odot}$ .

MACHOs.



Bullet Cluster (1E0657-558) -- Colliding clusters

1. 40 galaxies
2.  $z=0.296, D=1.41$  Gpc
3.  $kT=17.4$  keV,  $L_x=1.4 \times 10^{46}$  erg/s

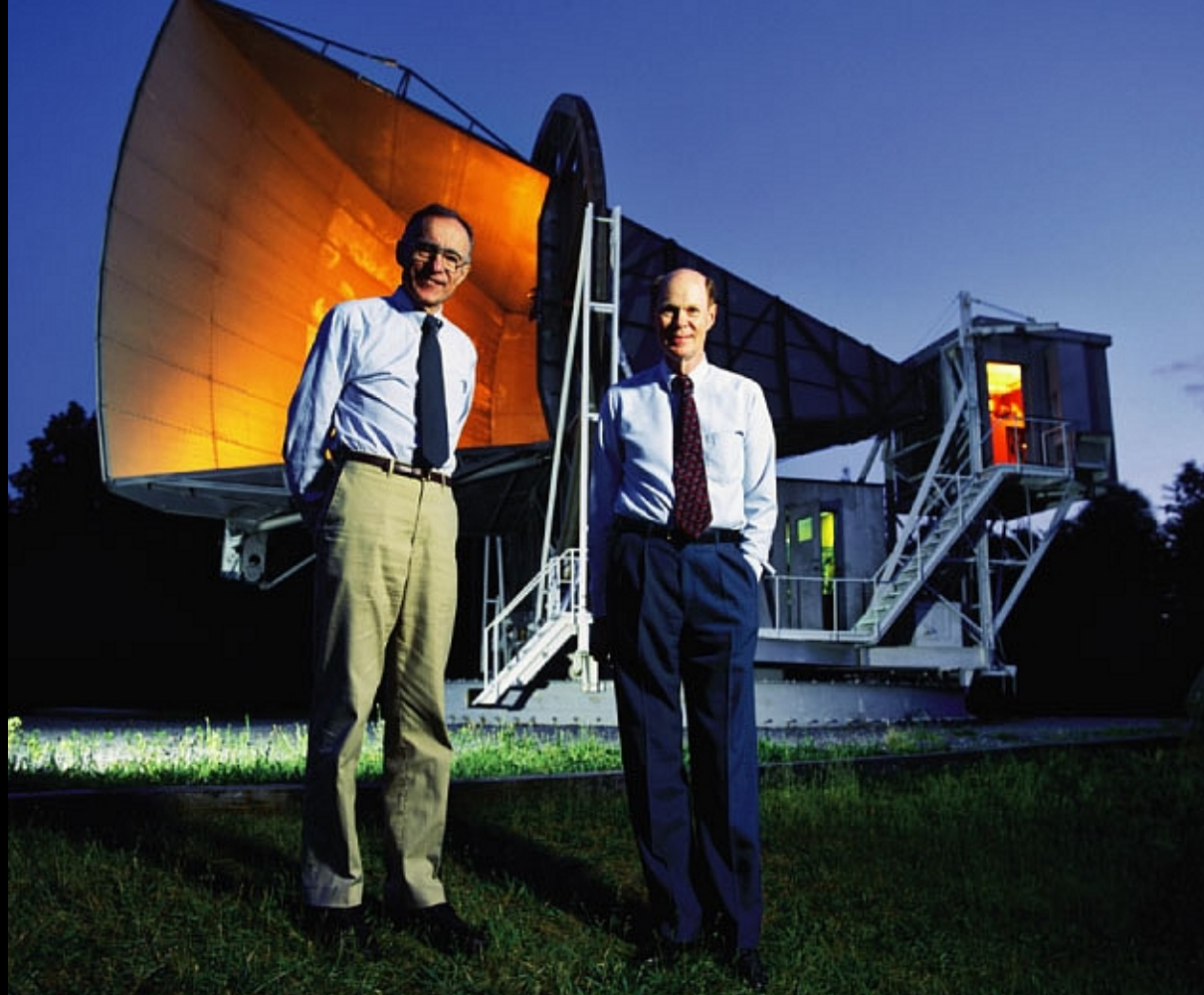


On the left is the dark matter distribution as determined by gravitational lensing

On the right is the x-ray image obtained by the Chandra X-ray Observatory with the mass contours superimposed.

Inferred from the Chandra observations of 1E0657-56, the cluster is undergoing a high-velocity (around 4500 km/s) merger, based on the distribution of the hot, X-ray emitting gas. The dark matter clump, revealed by the weak-lensing map, is coincident with the collisionless galaxies, but lies ahead of the collisional gas.

The velocity of the bullet subcluster is exceptionally high for a cluster substructure, and “cannot be accommodated within the currently favoured Lambda-CDM model cosmology.” A later study said the velocities of the collision as currently measured are “compatible with the prediction of a LCDM model.” Subsequent work found the collision to be consistent with LCDM simulations. The discrepancies resulted from small simulations and the methodology of identifying pairs. Earlier claims the Bullet Cluster was inconsistent with standard cosmology was based on an erroneous estimate of the speed of the shock in the X-ray emitting gas.

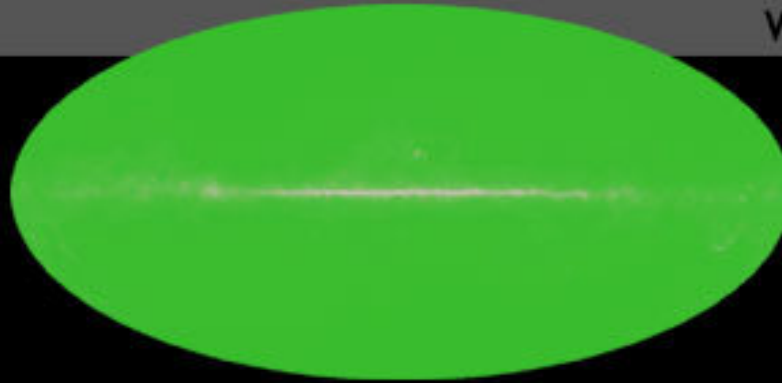


# COSMIC MICROWAVE BACKGROUND RADIATION (CMB)

1965



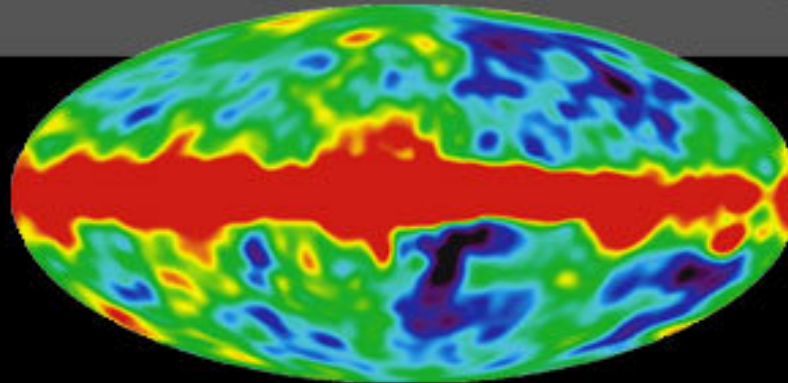
Penzias and  
Wilson



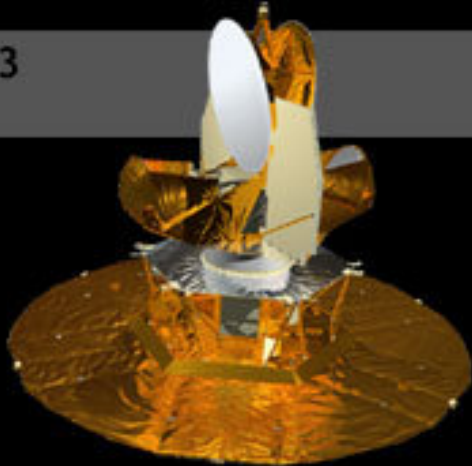
1992



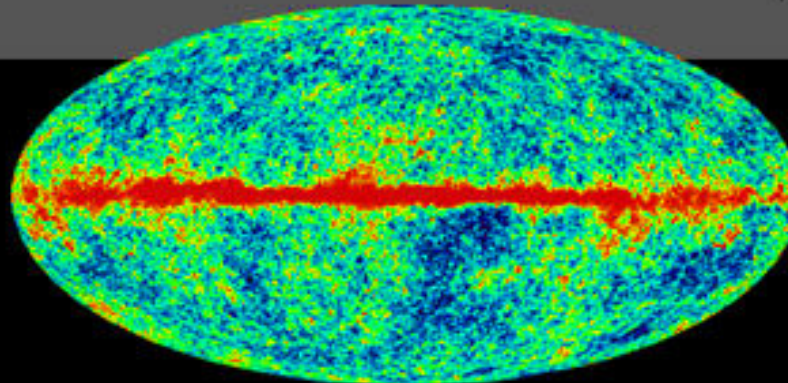
COBE



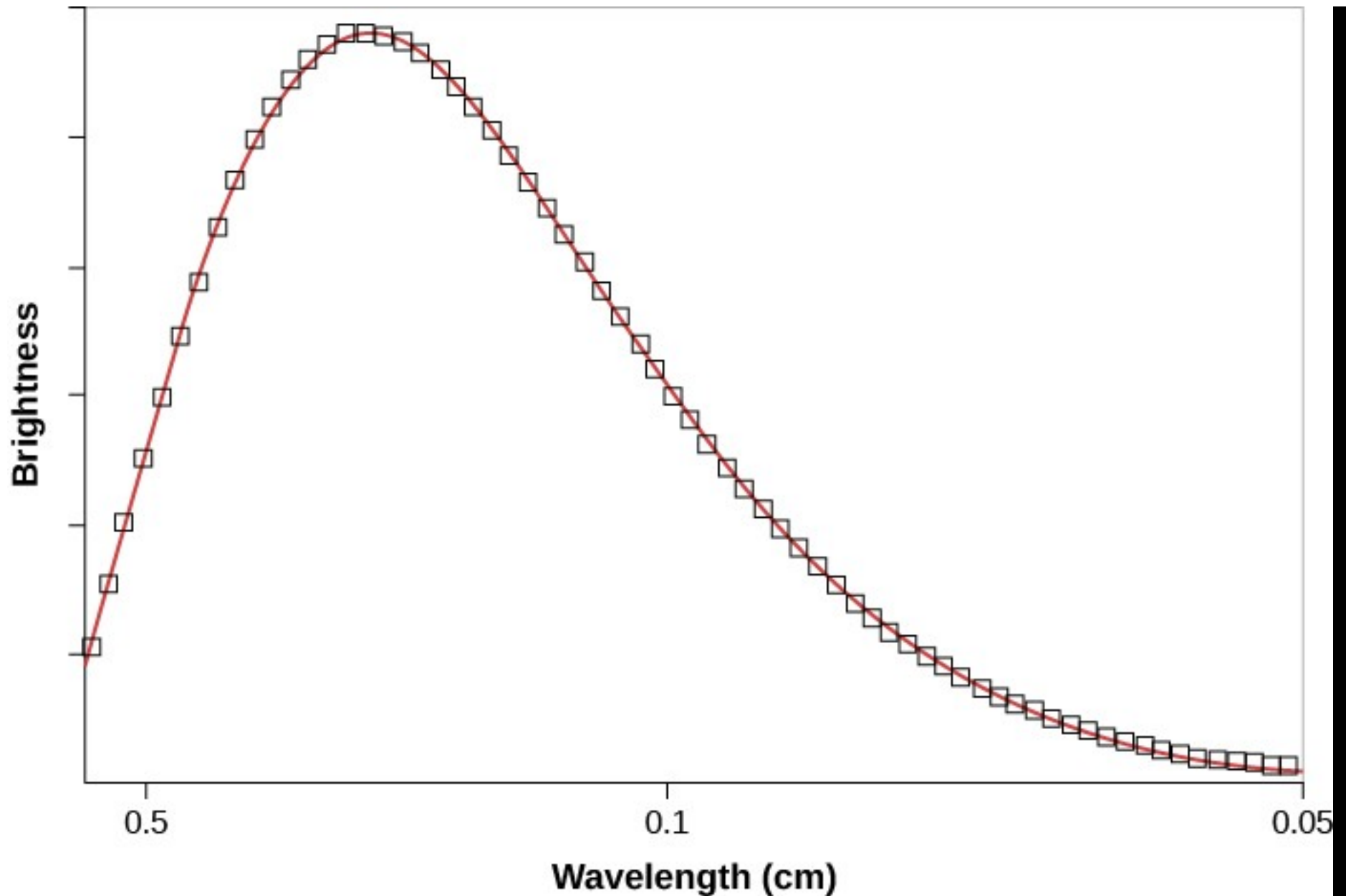
2003



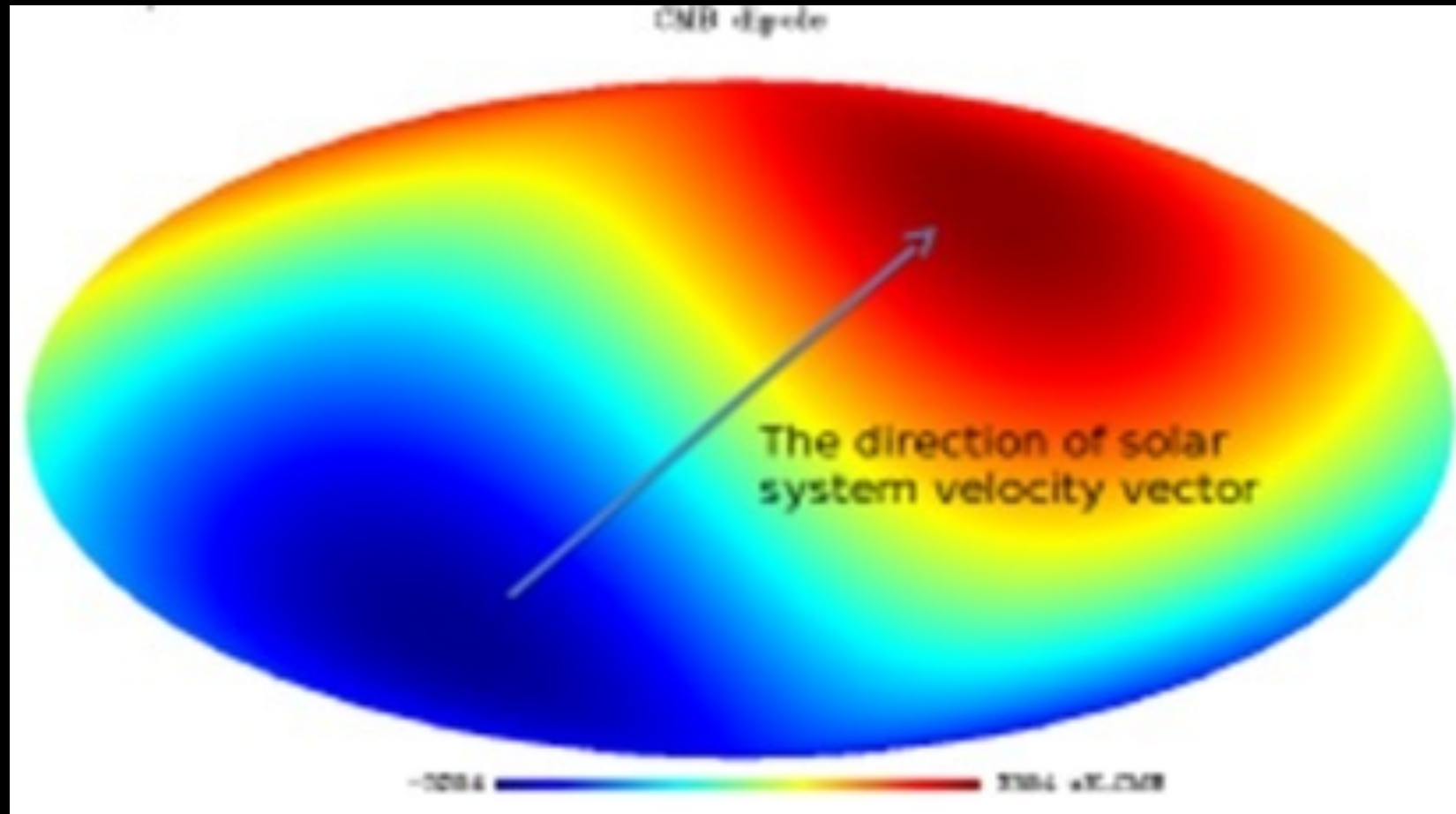
WMAP



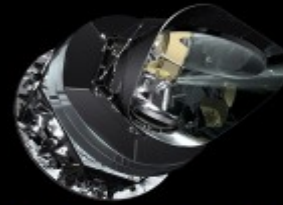
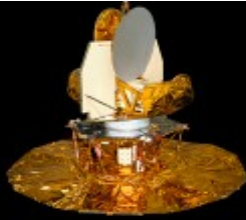




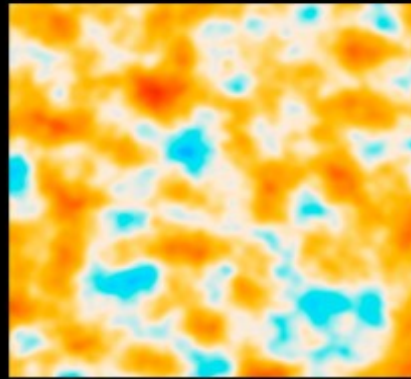
The spectrum of the CMBR is blackbody in shape with temperature  $T = 2.725$  K and is nearly isotropic except for some well-known distortions.



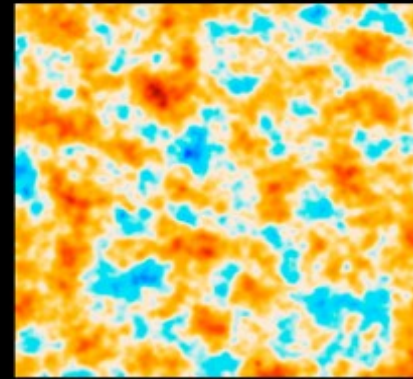
Dipole Anisotropy,  $\delta T = 0.0035 \text{ K} \rightarrow v = 0.002c$  in direction of Hydra



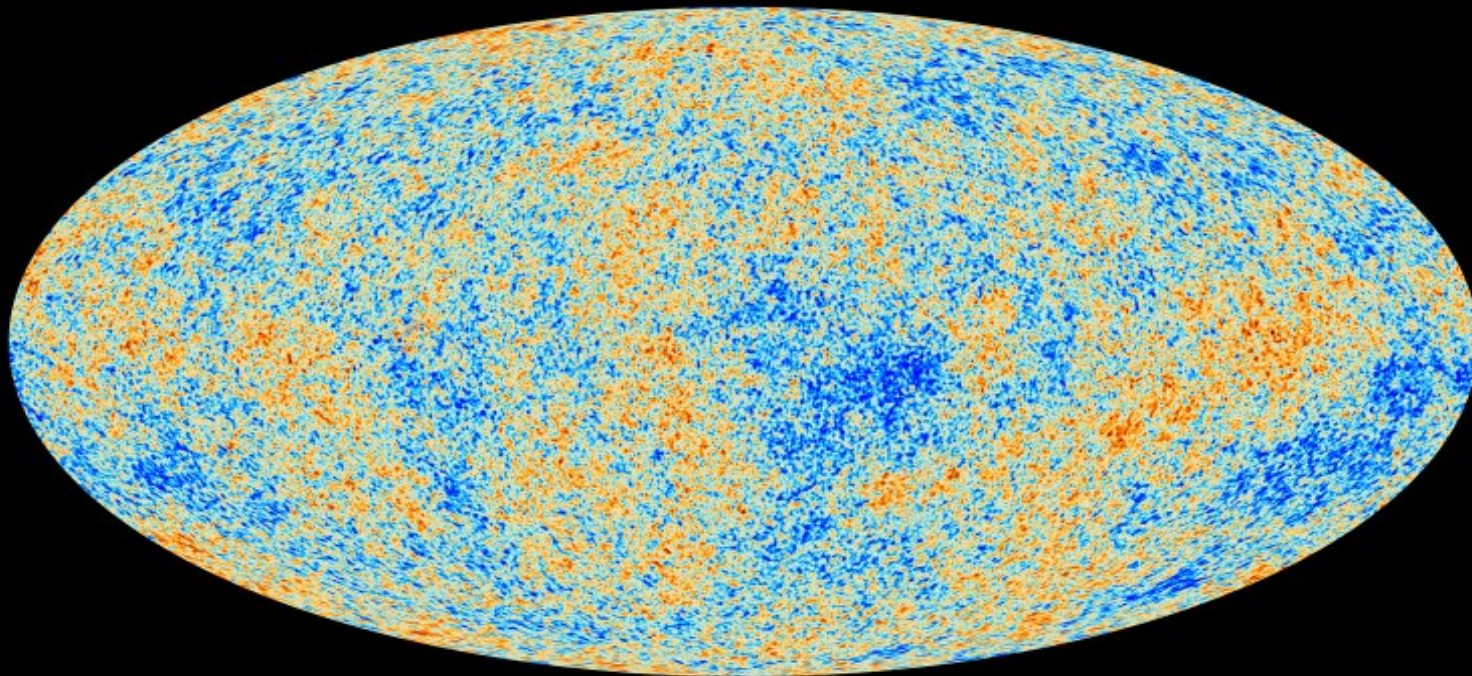
COBE



WMAP



Planck

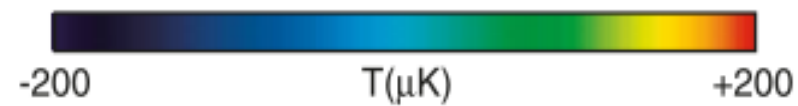
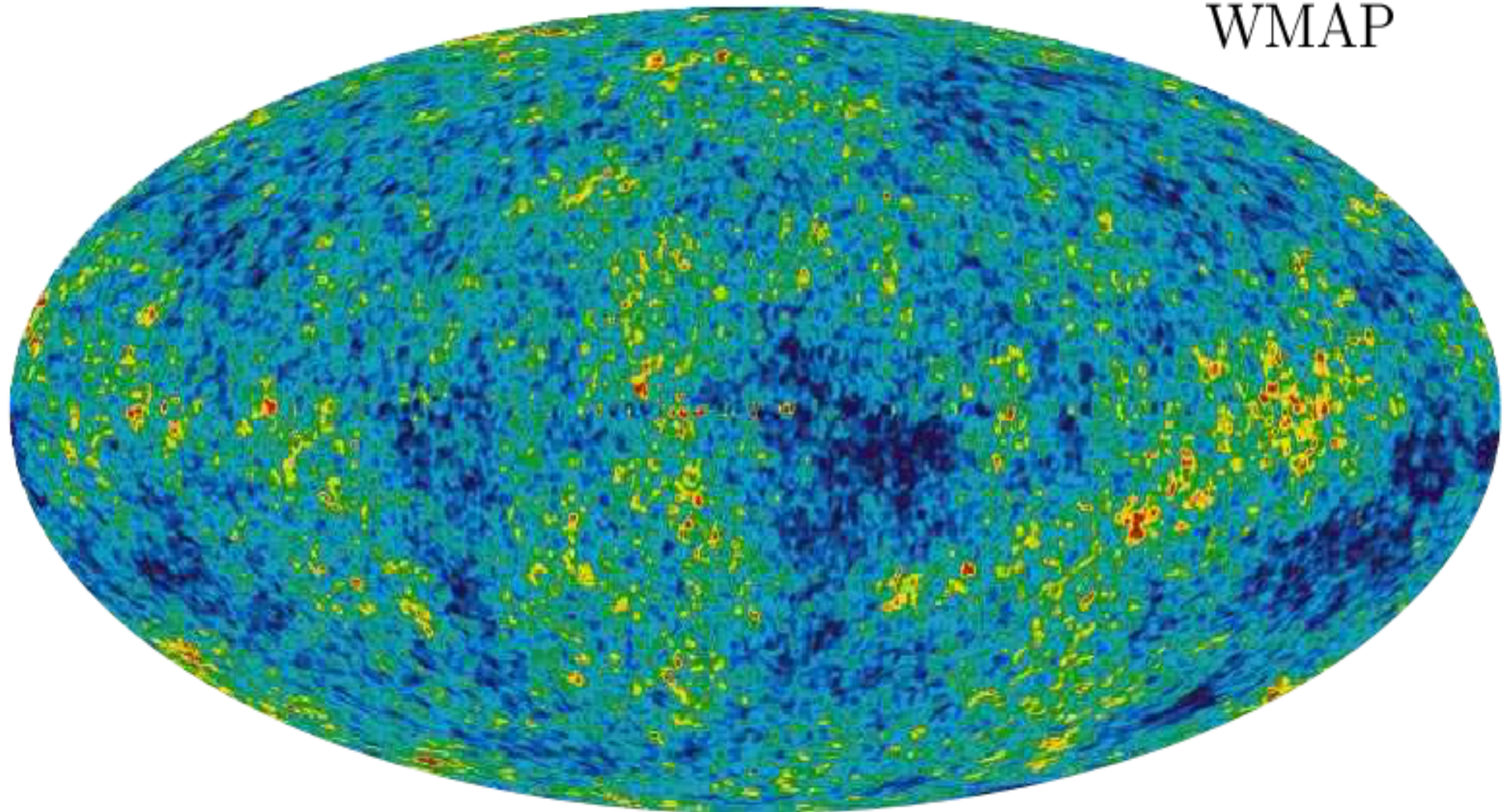


Quantum fluctuations

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$$T = 2.725^{\circ}\text{K}, \quad \frac{\delta T}{T} \sim 10^{-5}$$

WMAP

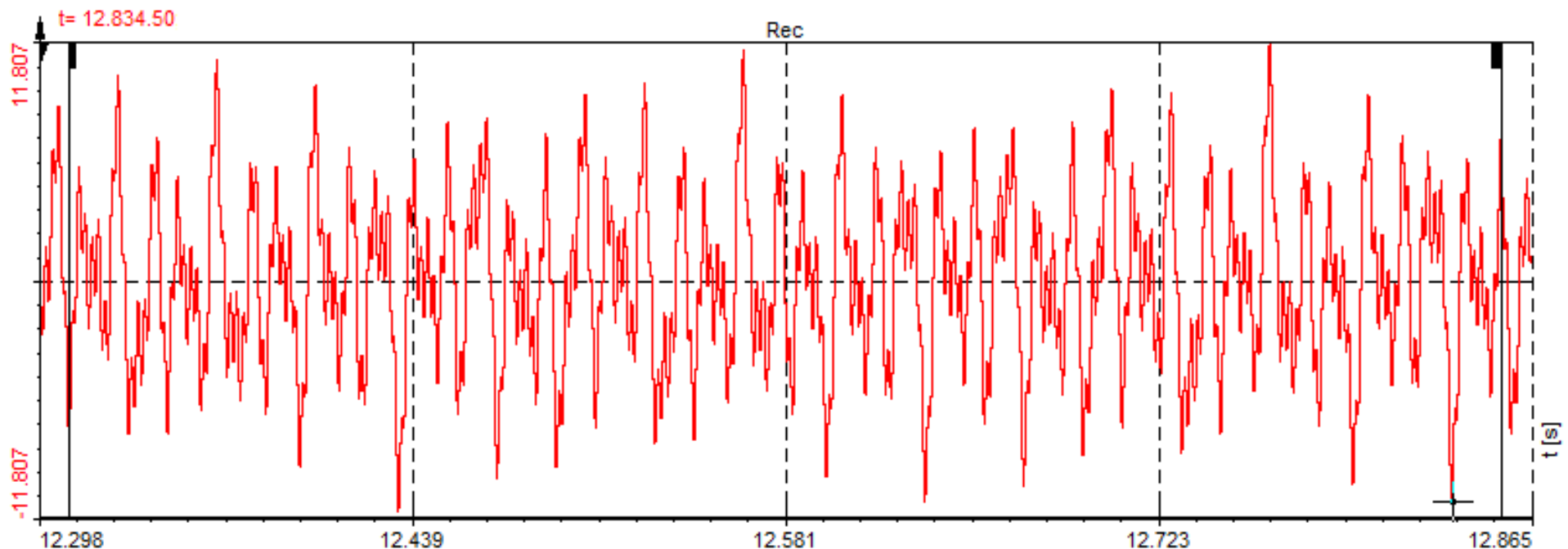


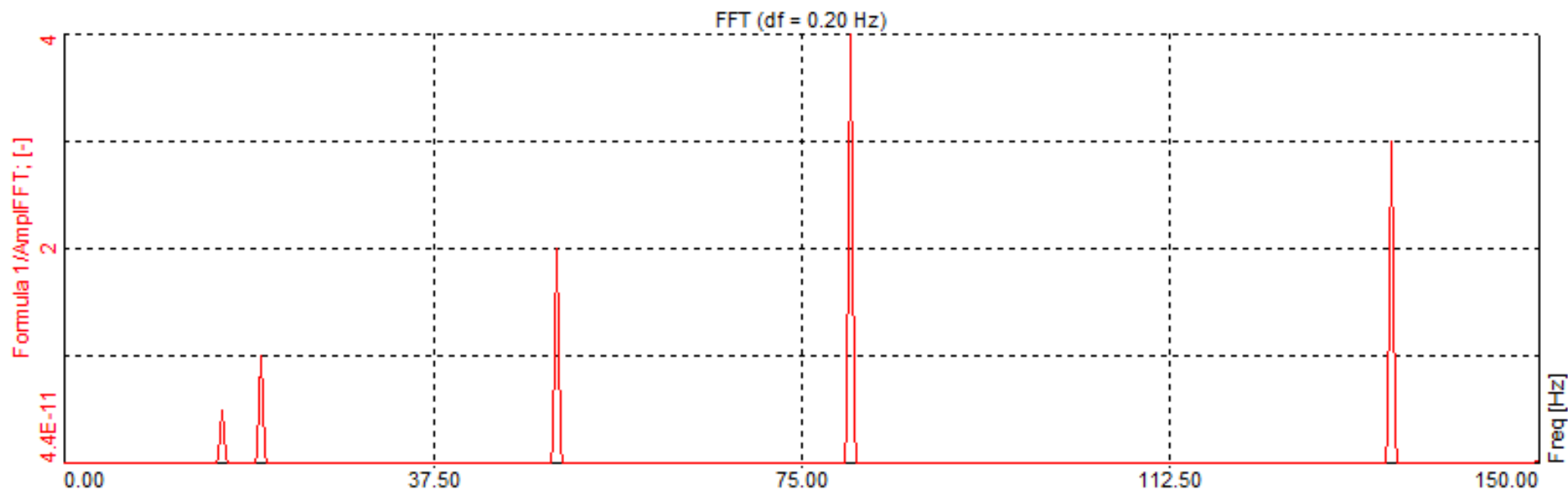
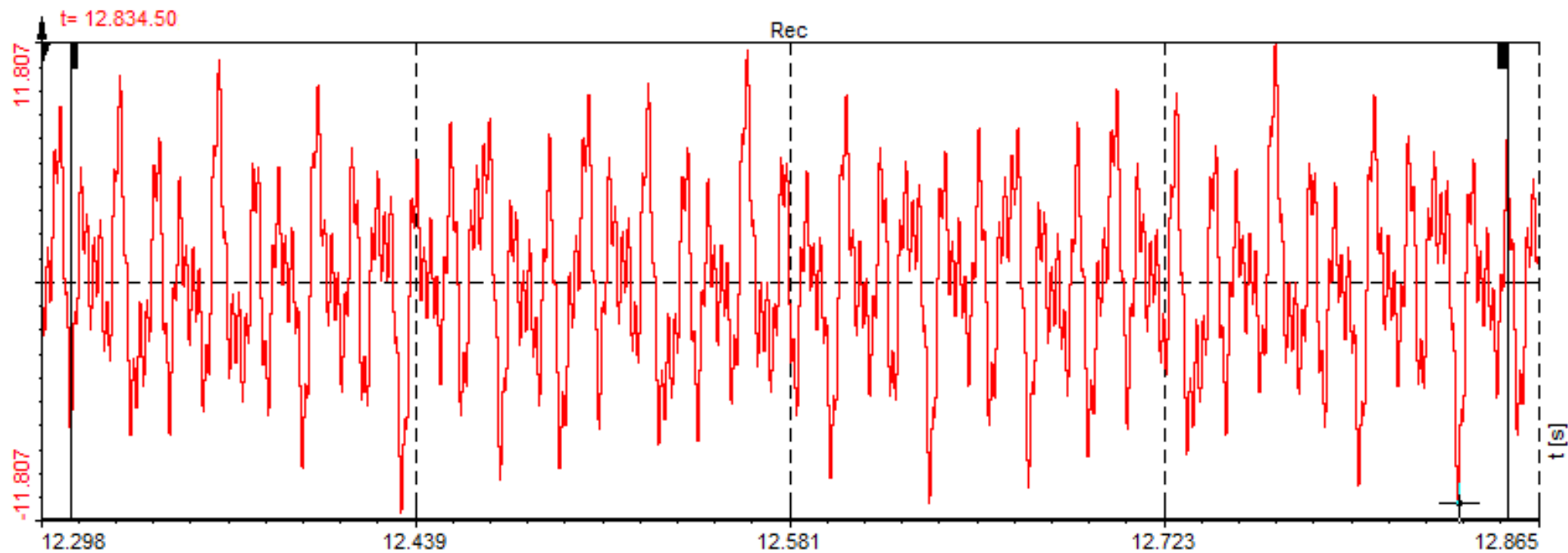
# Fourier Analysis

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

- Fitting a Fourier series to long strings of unevenly spaced data is straightforward, but inefficient from a computational standpoint. The work scales as  $N^2$  where  $N$  is the number of data points
- When the data points are evenly spaced, the numerical problem has a clever solution, the so-called the **FAST FOURIER TRANSFORM (FFT)** which, IMO, is the computer algorithm of the last 100 years. The FFT scales as  $N \log_e N \ll N^2$  for large  $N$ .
- Application of the **FFT** to the data below allows a rapid decomposition of the time series showing that a seemingly random process is actually a linear combination of a small number of waves

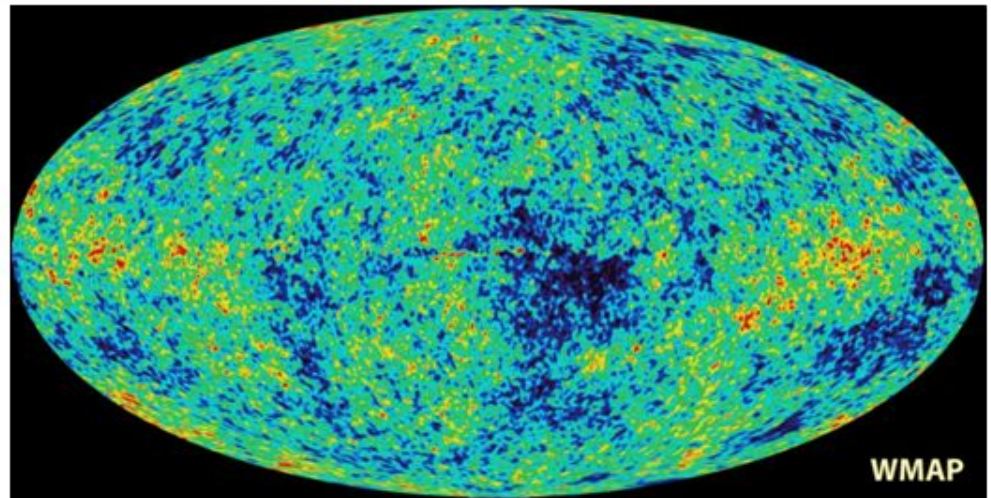




# Power Spectrum of CMB Temperature Fluctuations

Sky map of CMBR temperature fluctuations

$$\Delta(\theta, \varphi) = \frac{T(\theta, \varphi) - \langle T \rangle}{\langle T \rangle}$$

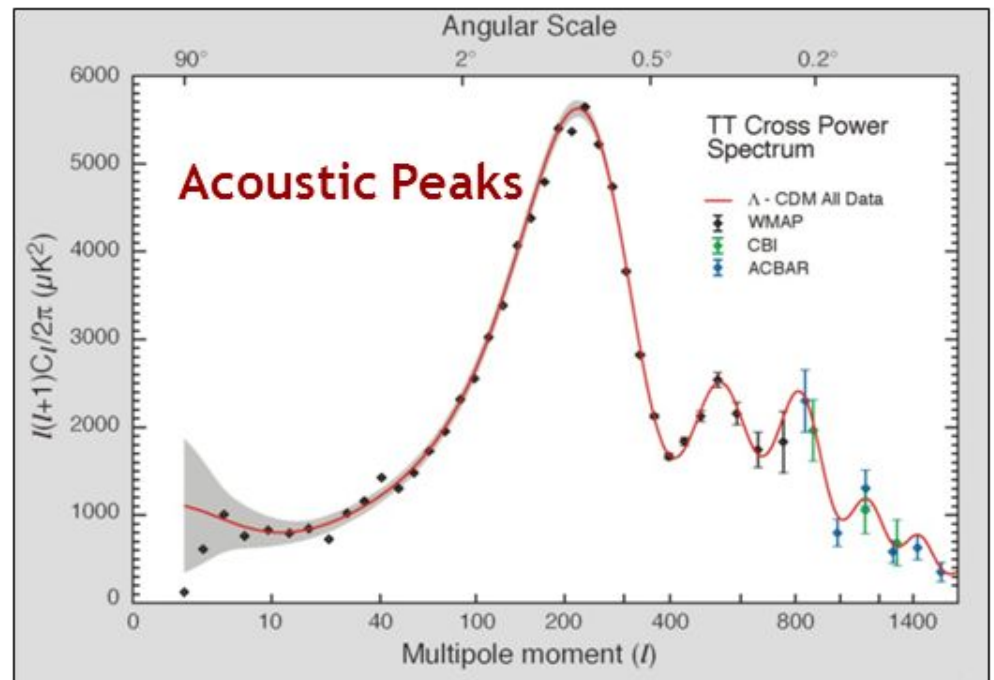


Multipole expansion

$$\Delta(\theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

Angular power spectrum

$$C_{\ell} = \langle a_{\ell m}^* a_{\ell m} \rangle = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} a_{\ell m}^* a_{\ell m}$$





# SPHERICAL HARMONICS

The spherical harmonics are given by

$$Y_{lm}(\theta, \varphi) = (-1)^m \left[ \frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!} \right]^{\frac{1}{2}} P_l^m(\cos \theta) e^{im\varphi}$$

For  $m \geq 0$ , and

$$Y_{l,-m} = (-1)^m Y_{lm}^*$$

The functions  $P_l^m$  are the associated Legendre polynomials defined by

$$P_l^m(u) = (-1)^{l+m} \frac{(l+m)!}{(l-m)!} \frac{(1-u^2)^{-m/2}}{2^l l!} \left( \frac{d}{du} \right)^{l-m} (1-u^2)^l$$

which is valid for  $m \geq 0$ , and the values for negative  $m$  given by

$$P_l^{-m}(u) = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(u)$$

(Remember  $u = \cos \theta$ ).

Determine the values of

# SPHERICAL HARMONICS

**TABLE 6.5**

The First Few Spherical Harmonics,  $Y_j^m(\theta, \phi)$  <sup>a</sup>

$$Y_0^0 = \frac{1}{(4\pi)^{1/2}}$$

$$Y_1^1 = -\left(\frac{3}{8\pi}\right)^{1/2} \sin \theta e^{i\phi}$$

$$Y_2^0 = \left(\frac{5}{16\pi}\right)^{1/2} (3 \cos^2 \theta - 1)$$

$$Y_2^{-1} = \left(\frac{15}{8\pi}\right)^{1/2} \sin \theta \cos \theta e^{-i\phi}$$

$$Y_2^{-2} = \left(\frac{15}{32\pi}\right)^{1/2} \sin^2 \theta e^{-2i\phi}$$

$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos \theta$$

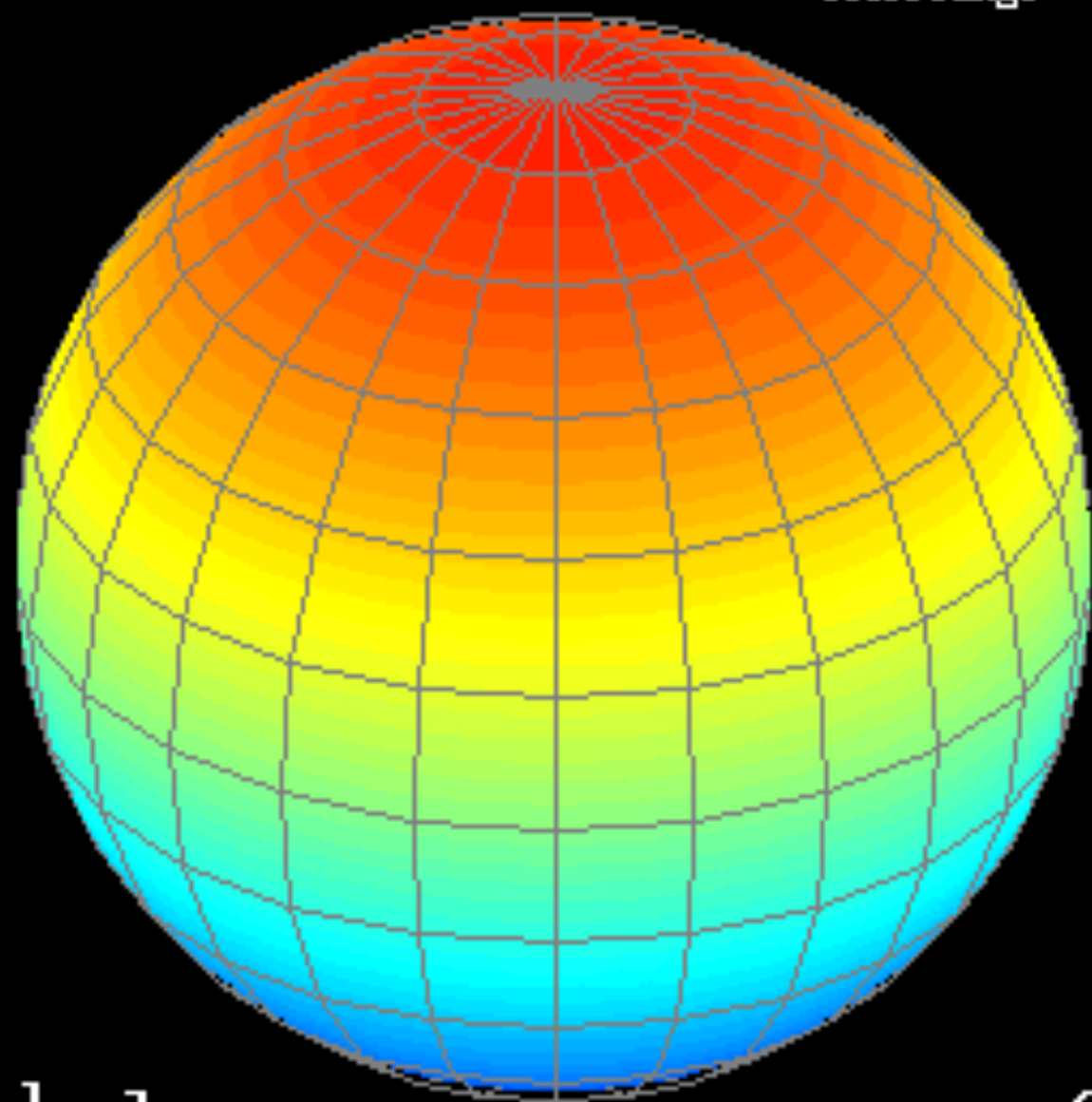
$$Y_1^{-1} = \left(\frac{3}{8\pi}\right)^{1/2} \sin \theta e^{-i\phi}$$

$$Y_2^1 = -\left(\frac{15}{8\pi}\right)^{1/2} \sin \theta \cos \theta e^{i\phi}$$

$$Y_2^2 = \left(\frac{15}{32\pi}\right)^{1/2} \sin^2 \theta e^{2i\phi}$$

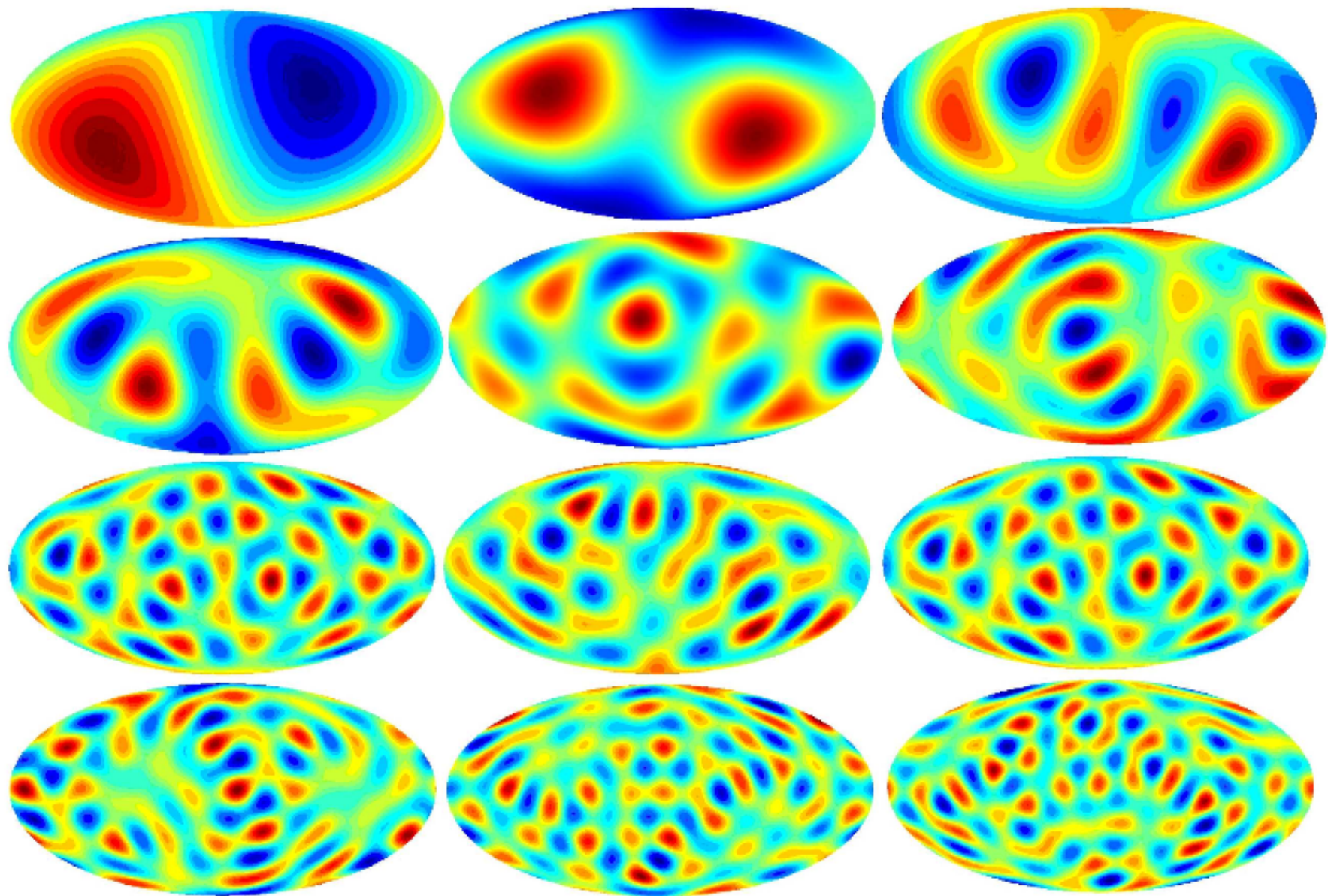
a. The negative signs in  $Y_1^1(\theta, \phi)$  and  $Y_2^1(\theta, \phi)$  are simply a convention.

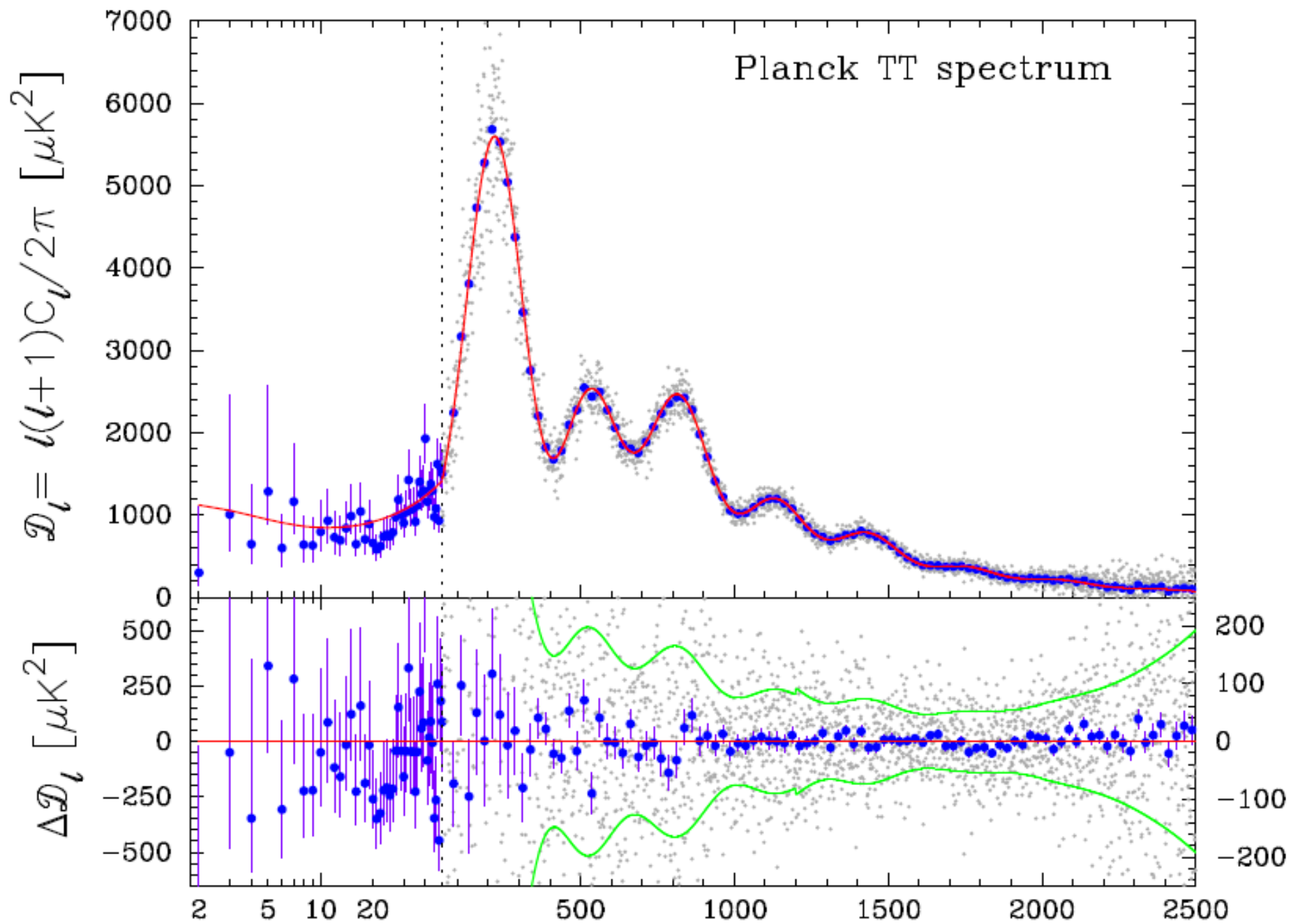
color range= $\pm 0.69$

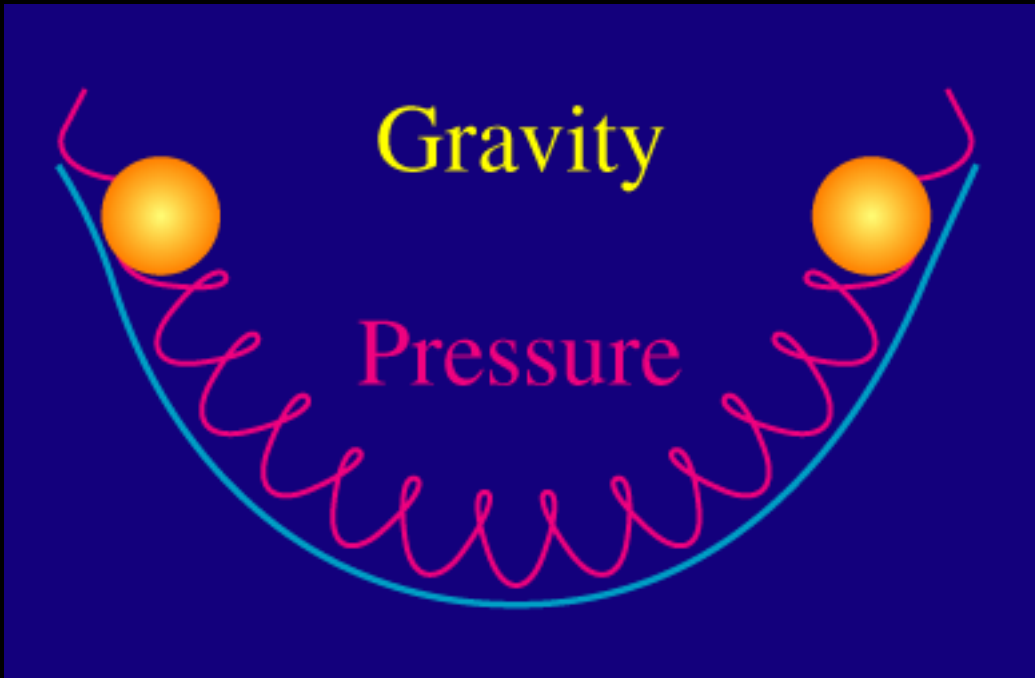


$l=1$

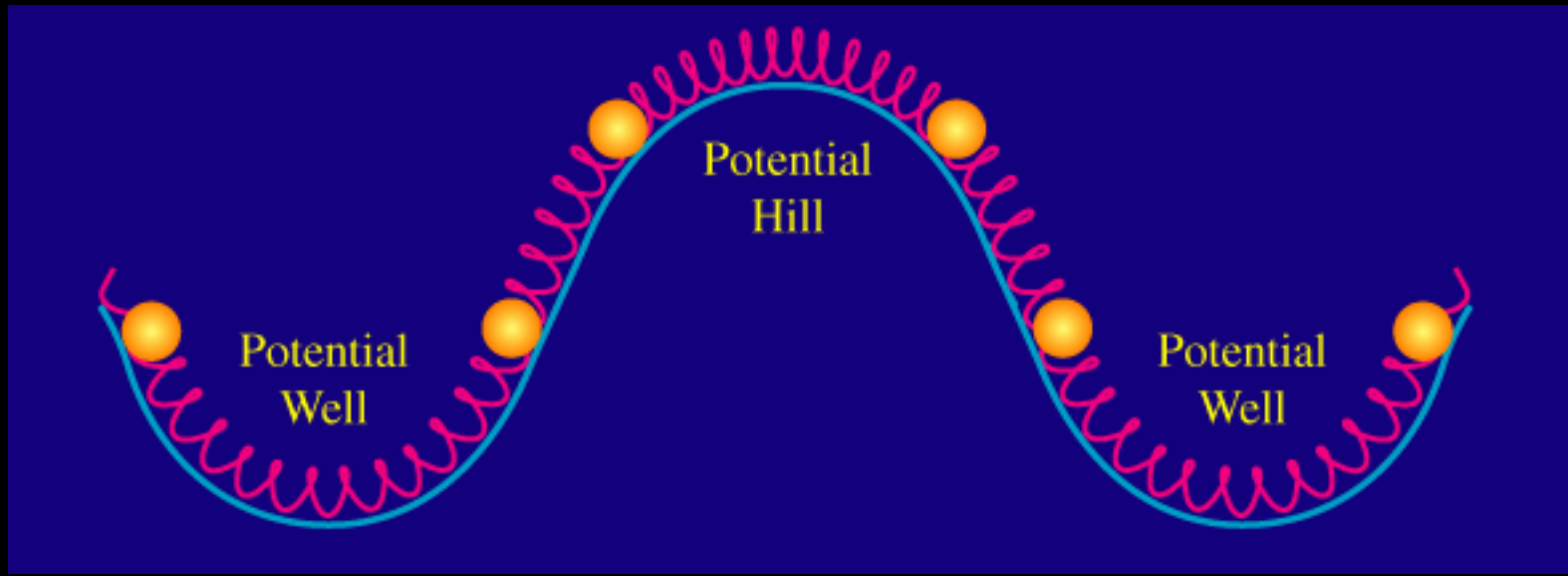
$m=0$

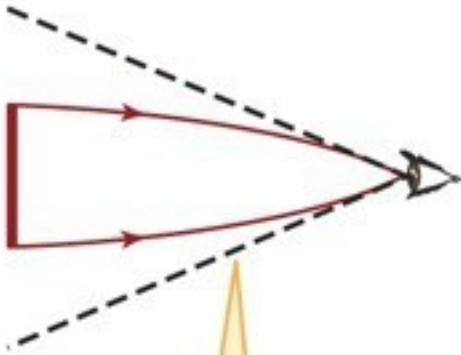




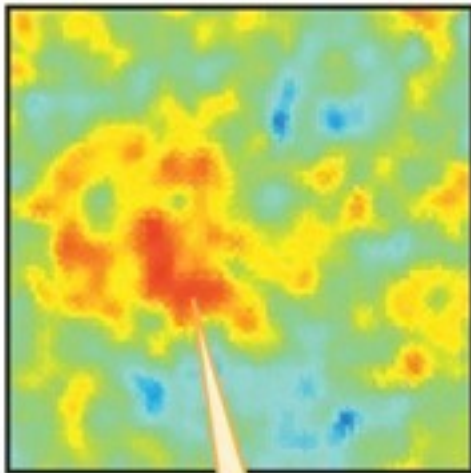


Dark Matter,  
Baryons,  
And Photons





If the universe is closed, light rays from opposite sides of a hot spot bend toward each other ...

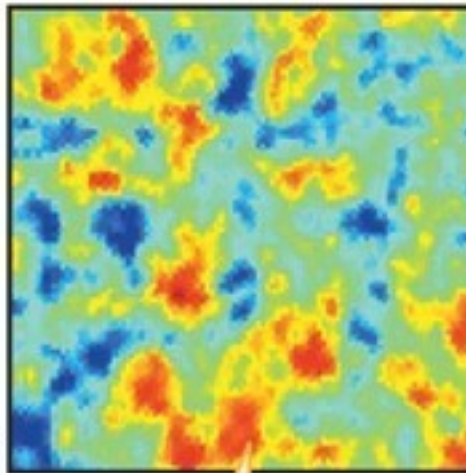


(a)

... and as a result, the hot spot appears to us to be larger than it actually is.



If the universe is flat, light rays from opposite sides of a hot spot do not bend at all ...

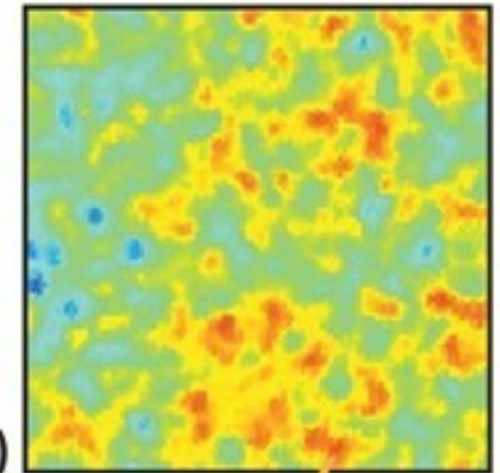


(b)

... and so the hot spot appears to us with its true size.

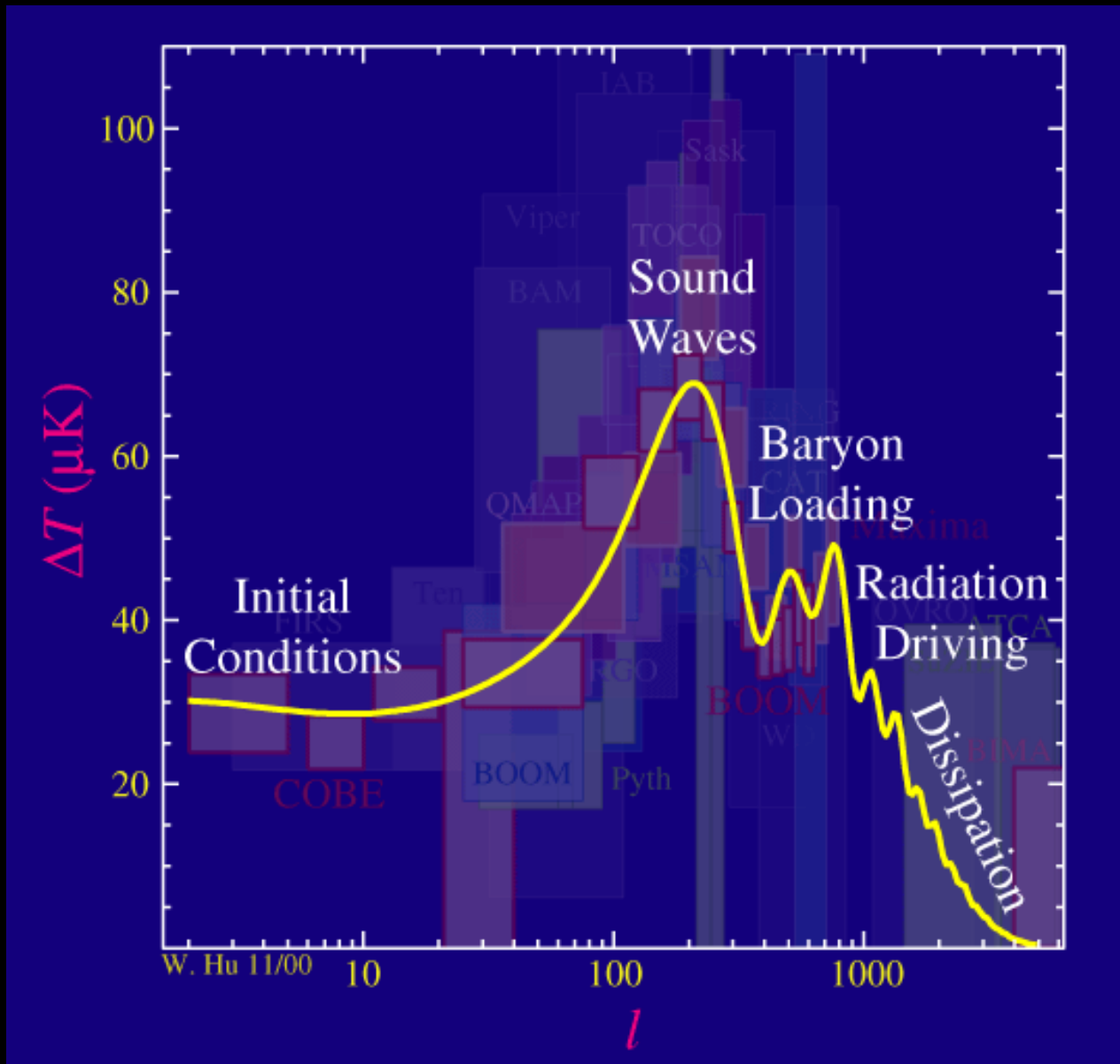


If the universe is open, light rays from opposite sides of a hot spot bend away from each other ...



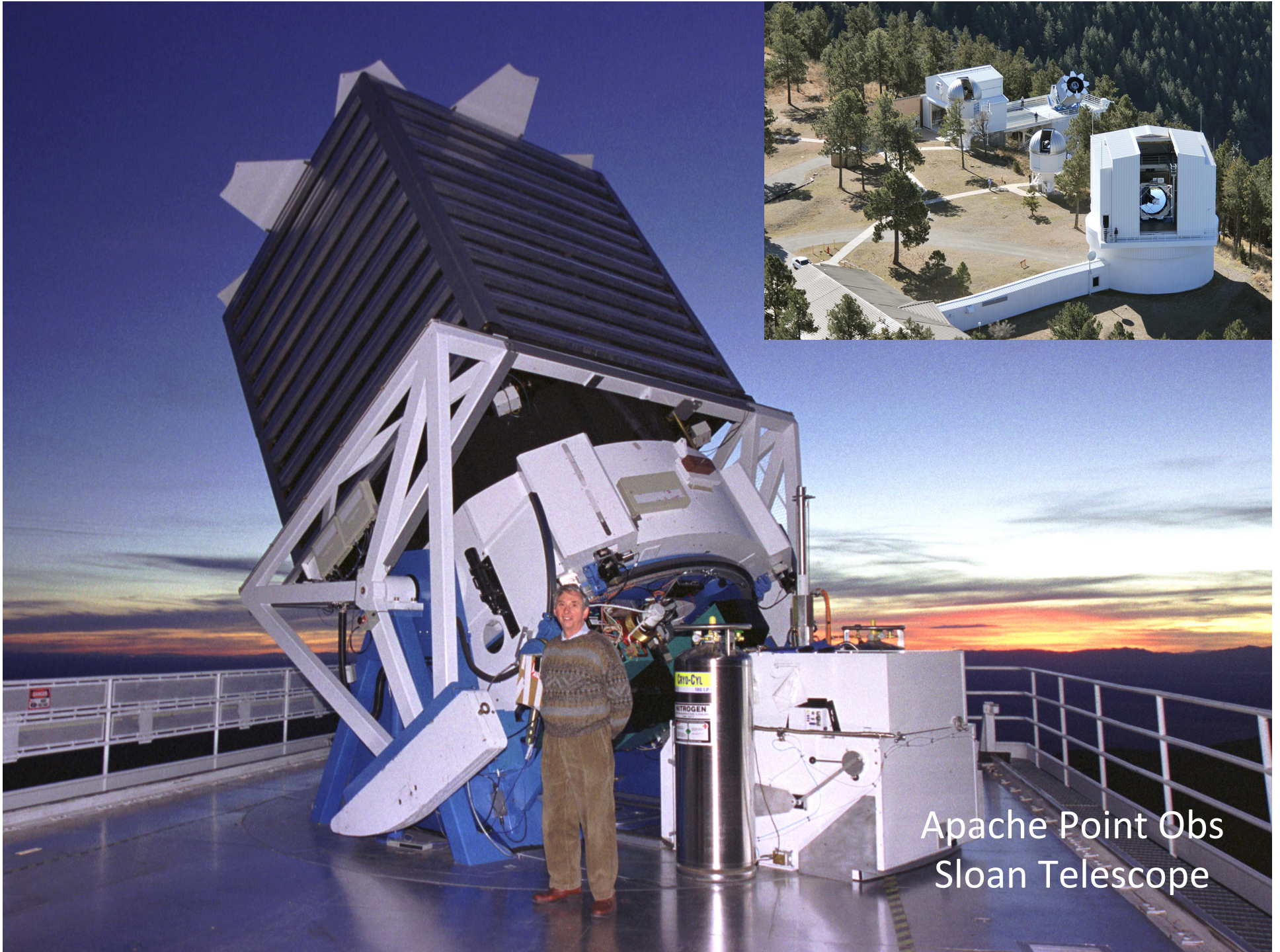
(c)

... and as a result, the hot spot appears to us to be smaller than it actually is.

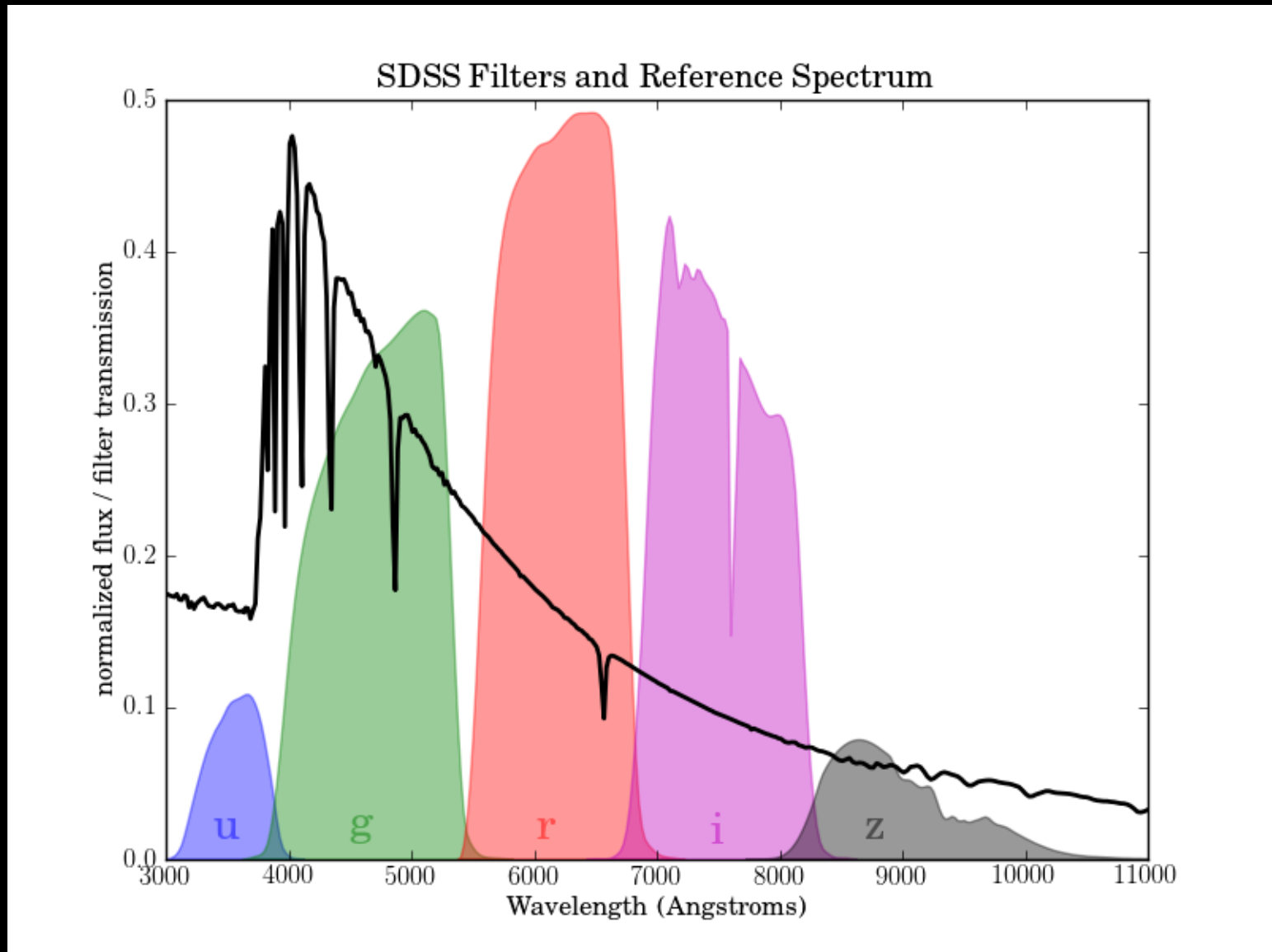


Acoustic Peaks





Apache Point Obs  
Sloan Telescope



Sloan Filter Set

# Two-Point Correlation Functions

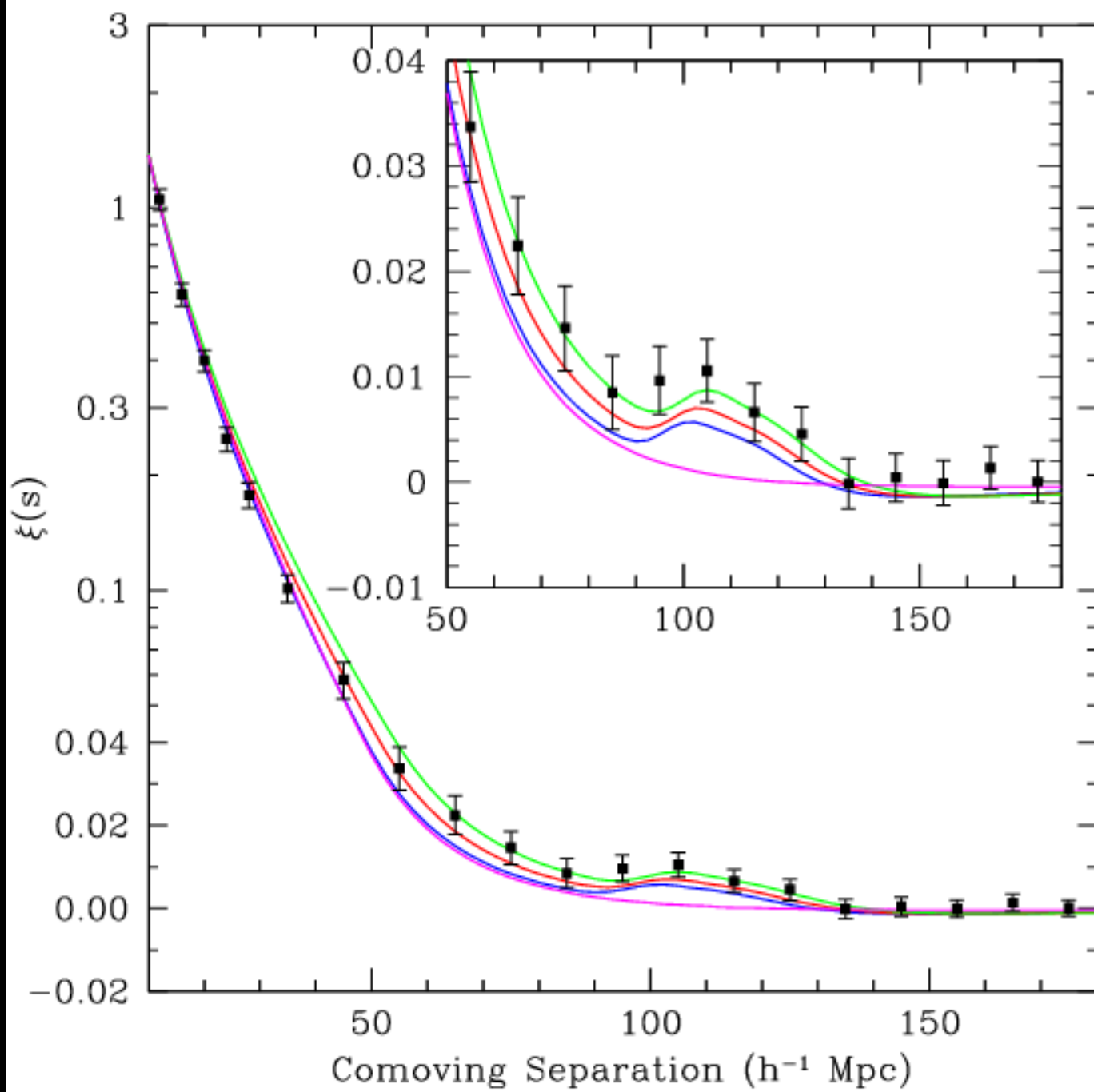
Local density fluctuations are given by

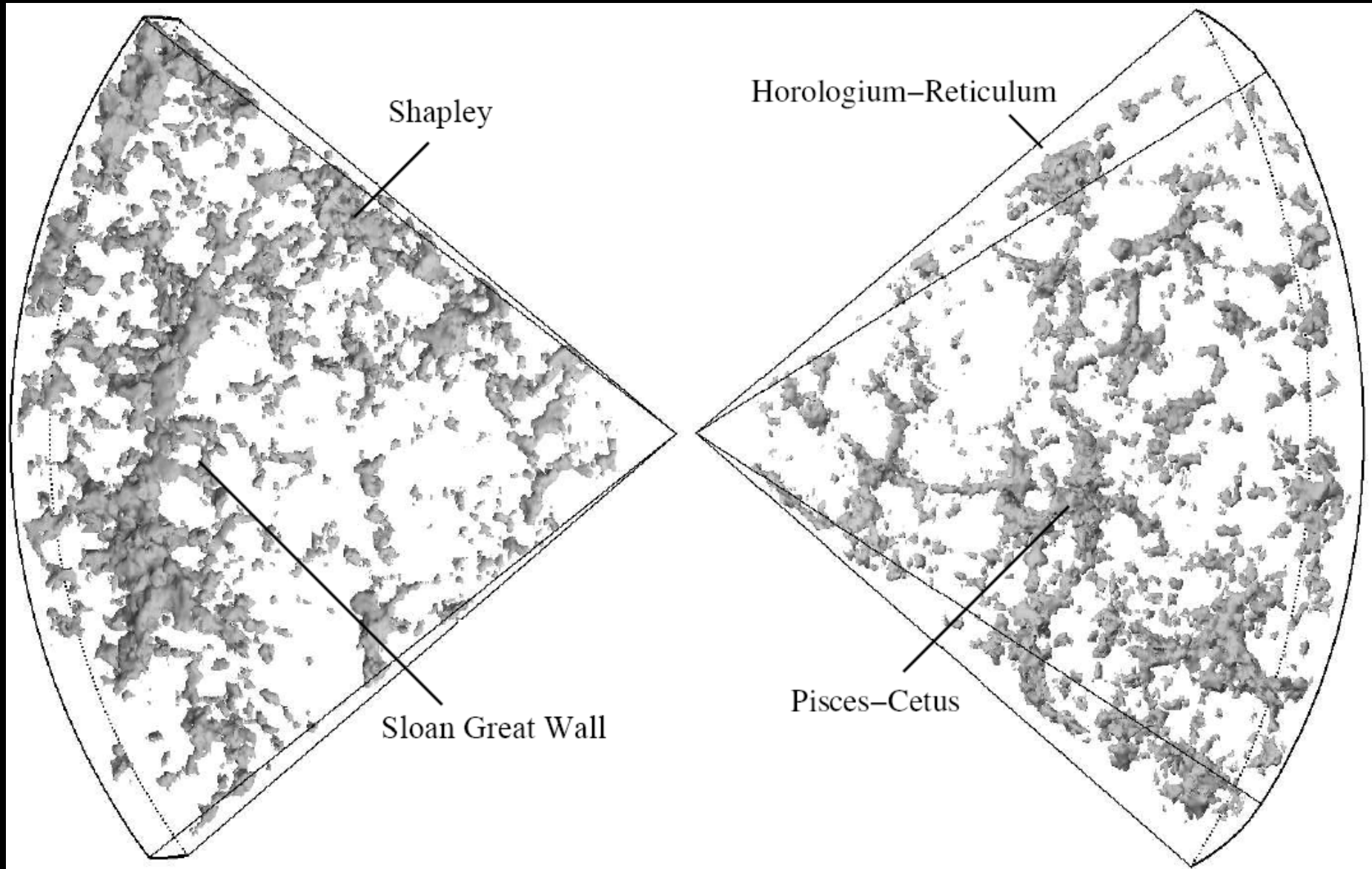
$$\delta(x) = [\rho(x) - \rho_0(x)] / \rho_0(x),$$

where  $\rho(x)$  is the local density and  $\rho_0(x)$  is the average density. The two-point correlation function is then

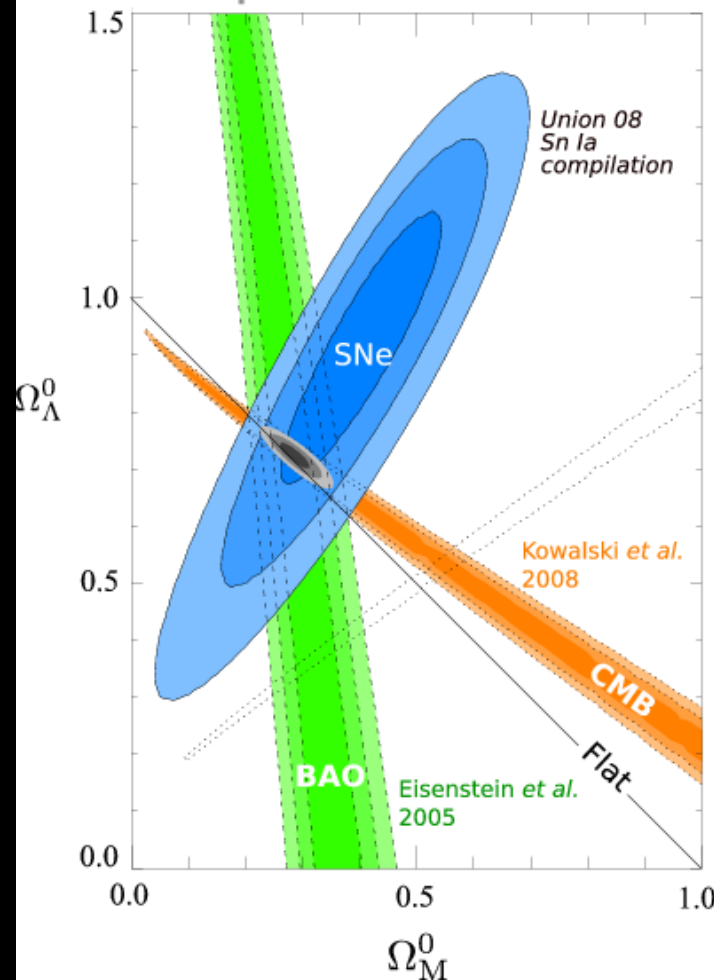
$$\xi(x_1, x_2) = \langle \delta(x_1) \cdot \delta(x_2) \rangle$$

where  $x_1$  and  $x_2$  are two spatial locations. If the fluctuations are uncorrelated then the expectations will be randomly distributed and the sums of  $\xi(x_1, x_2)$  for fixed distances,  $s$ , between  $x_1$  and  $x_2$  average to 0, no correlation. If there is correlation then  $\xi(s)$  may not be 0.



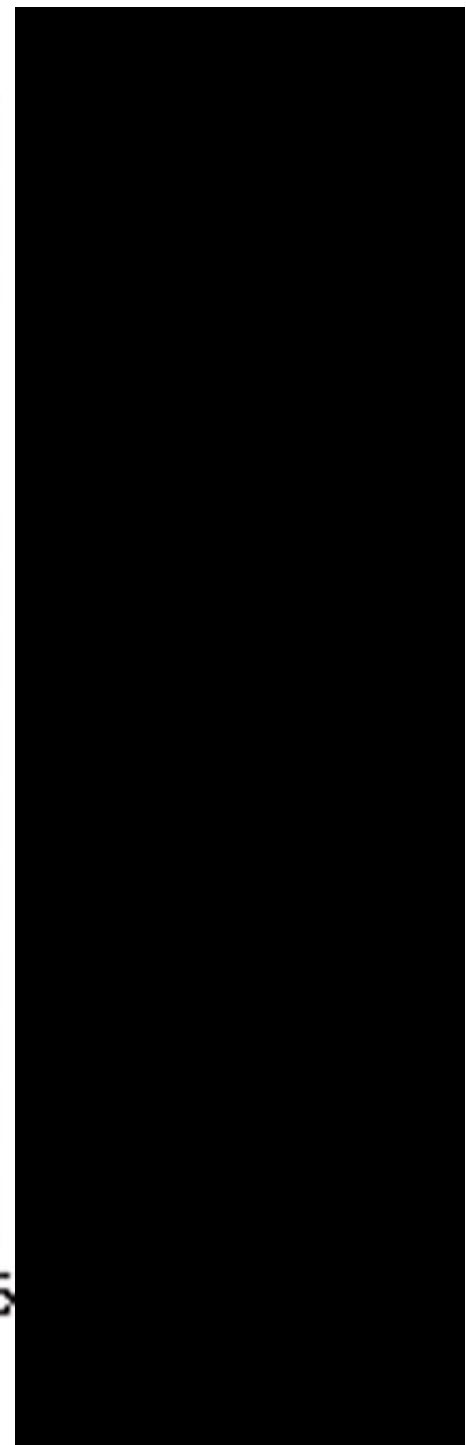
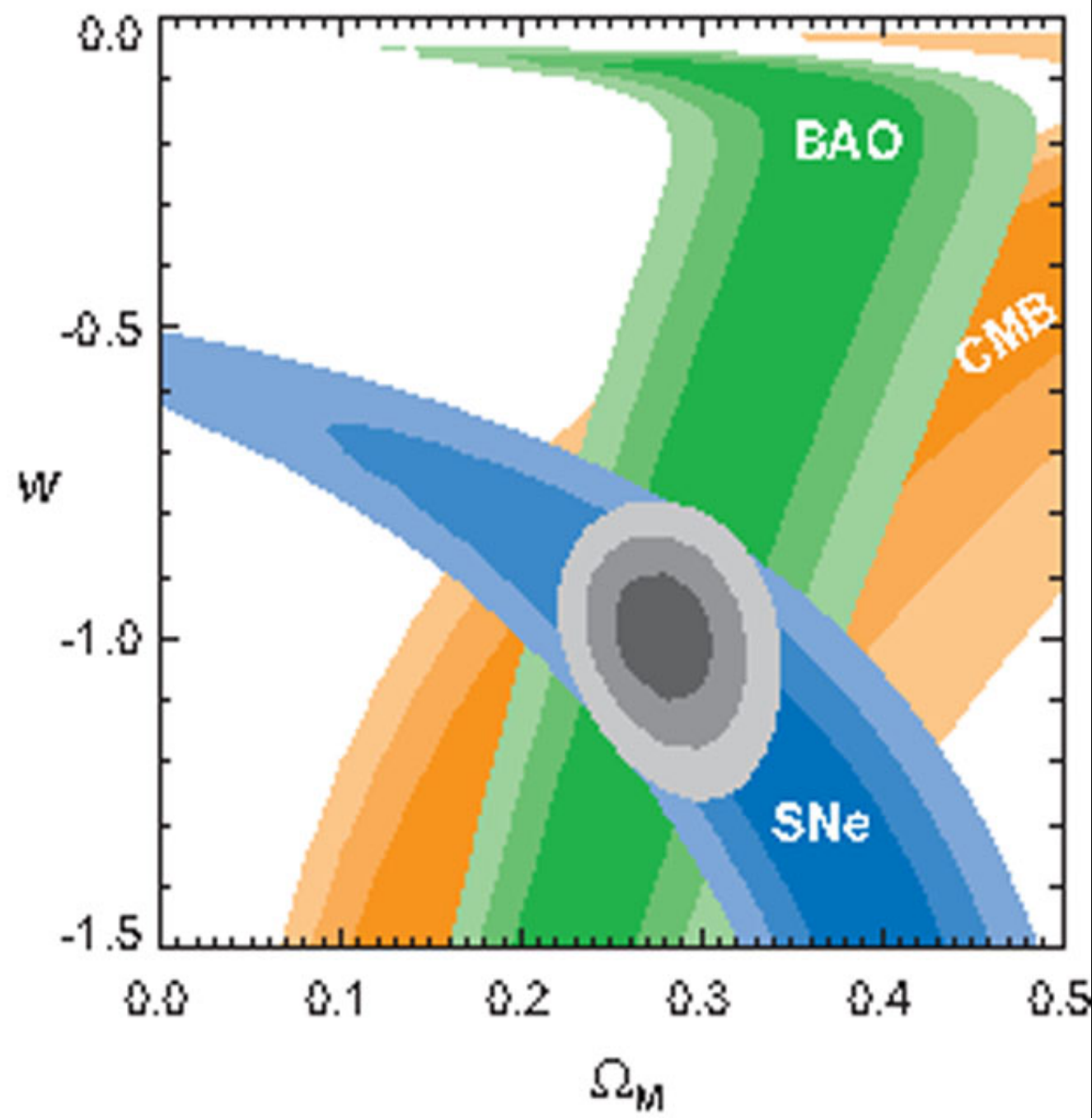


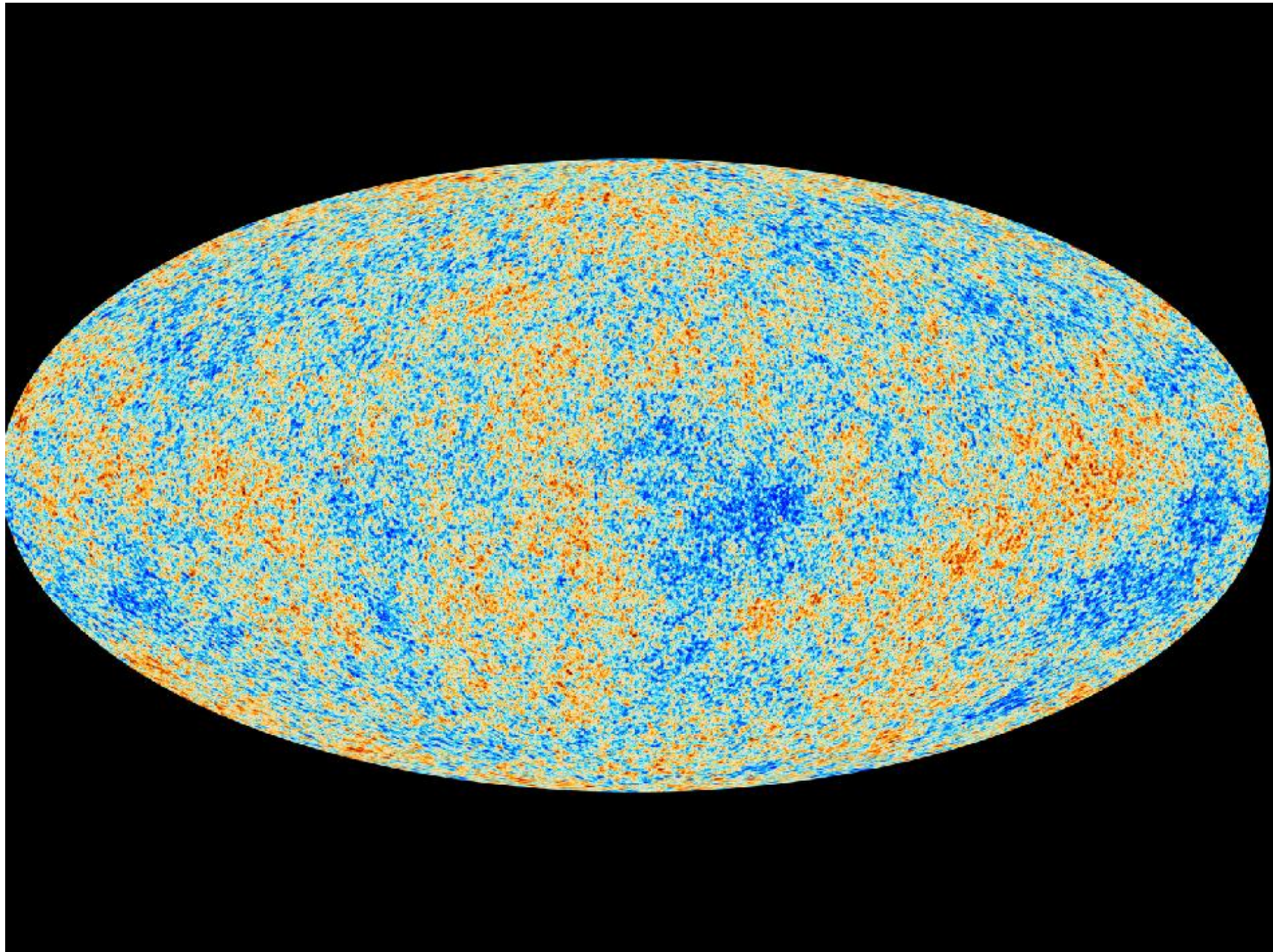
Supernova Cosmology Project  
Kowalski, et al., *Ap.J.* (2008)



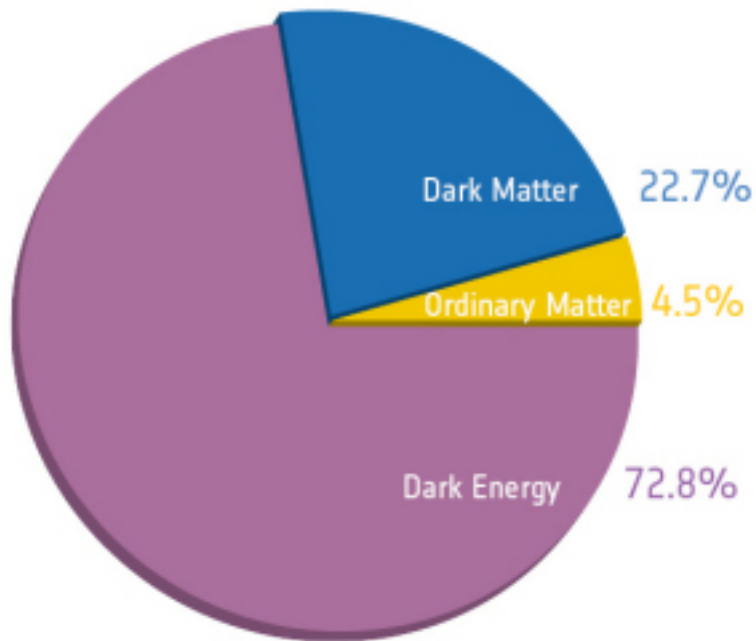
$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3} \bar{\rho} + \frac{\Lambda}{3} - \frac{K}{a^2}; \quad (K = 0, \pm 1), \quad (17)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \bar{\rho} + \frac{\Lambda}{3}, \quad (18)$$

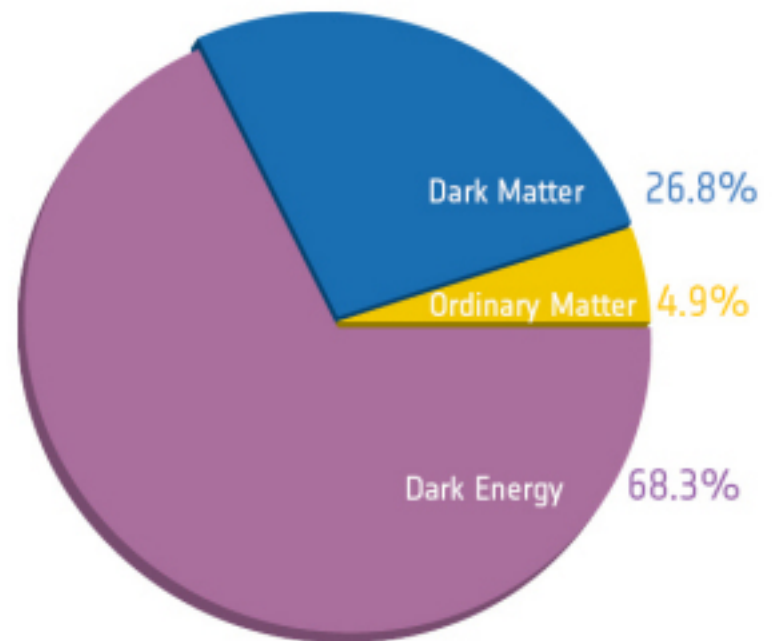








Before Planck



After Planck