Homeowrk 8
Due: June 9, 2015

45. Chapter 3, Section 11, Problem 48
46. Chapter 3, Section 11, Problem 62
47. The Hamiltonian operator for the one-dimensional simple harmonic oscillator is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}, \tag{1}$$

where $\hat{p}$ and $\hat{x}$ are operators, m is the mass, and ω is the oscillator frequency. Show that the Hamiltonian operator is Hermitian.
48. A special linear operator $|\alpha\rangle\langle\beta|$ is defined by its action on an arbitrary *ket*, $|\psi\rangle$,

$$(|\alpha\rangle\langle\beta|)|\psi\rangle = |\alpha\rangle\langle\beta|\psi\rangle. \tag{2}$$

Show that its action on an arbitrary *bra* is given by

$$\langle\psi|(|\alpha\rangle\langle\beta|) = \langle\psi|\alpha\rangle\langle\beta|. \tag{3}$$

49. and 50. If A is a linear operator, than $A^\dagger$ is another linear operator defined by its action on an arbitrary *ket*, $|\psi\rangle$,

$$A^\dagger|\psi\rangle = (\langle\psi|A)^\dagger. \tag{4}$$

Using equation [2], show that

$$\langle\phi|A^\dagger|\psi\rangle = (\langle\psi|A|\phi\rangle)^*, \tag{5}$$

$$(|\alpha\rangle\langle\beta|)^\dagger = |\beta\rangle\langle\alpha| \tag{6}$$