Homeowrk 1
Due: 2012 October 5

1. Problem 2.1
2. Problem 2.4
3. Problem 2.6
4. Problem 2.9
5. Problem 2.14
6. Find and sketch the field lines for an eletric dipole field.
7. Consider the infinetesimally thin flat disk of Problem 2.6. Set the symmetry axis of the disk to lie along the z-axis and assume that $\sigma > 0$.

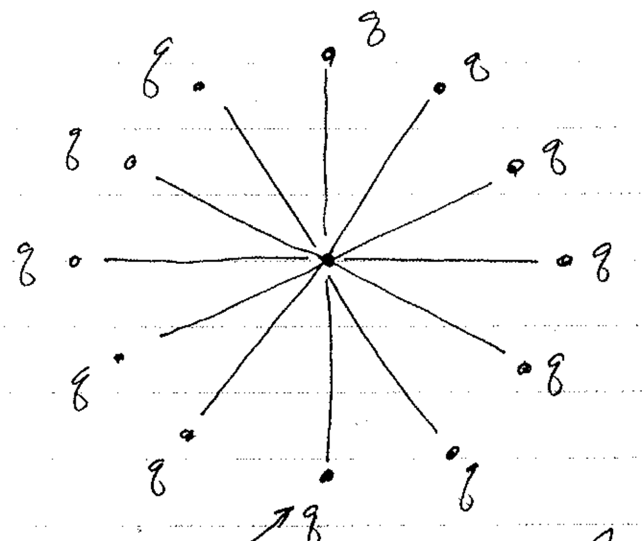
  a. A charge q, where $q\sigma < 0$, is placed at the center of the disk. What is the force on charge q?

  c. If charge q can move only along the symmetry axis of the disk, find and describe its motion if it is displaced a small height, $|\delta z| \ll R$, and then released.

Asst #1

① Prob 2.1

a) 12 equal charges q are placed at the corners of a 12-sided polygon. What is the net force placed on a charge Q at the center of the polygon?



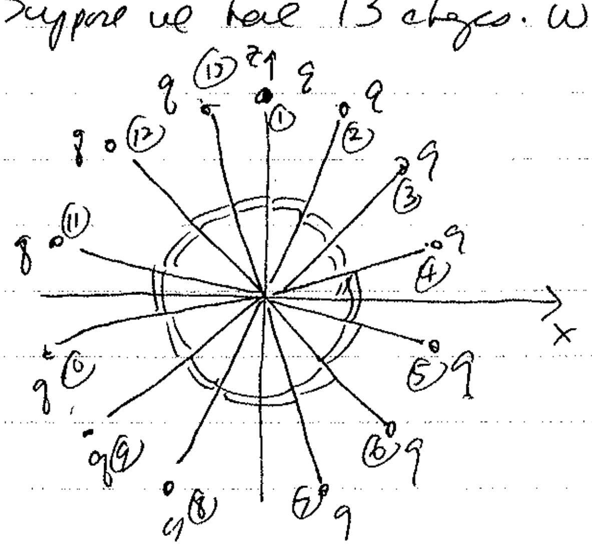
By symmetry, each charge q has an opposite partner, the net $\vec{E}$ at the center is 0

$$\rightarrow \vec{F}_a = 0$$

b) Suppose q is removed. What is the force on Q? If each charge is a distance R from the center, only the charge at 12 is unbalanced. If we define the z-axis as aligned from 6pm → 12 and we remove the charge at 12,

$$\rightarrow \vec{F}_a = \frac{(-1)}{4\pi\epsilon_0}\frac{qQ}{R^2}\hat{z}$$

c) Suppose we have 13 charges. What is the field at the center?



(i) Each charge is separated from its neighbor by

$$a = \frac{360°}{13}$$

ii) Each charge pair; (13, 2), (12, 3), (11, 4), (10, 5) (9, 6), (8, 7) cancels in the x-dir^n $\Rightarrow$ $\vec{E}_x = 0$ at the center. (charge (1) has no x-component).

$\Rightarrow$ the only possible field can lie in the z-dir^n

(iii) there was nothing special about using charge 1 as the "center". This argument holds true for any other charge.

$\Rightarrow$ $\vec{E}$ must be 0 at the origin.*

d) If charge at (1) is removed the force on Q becomes

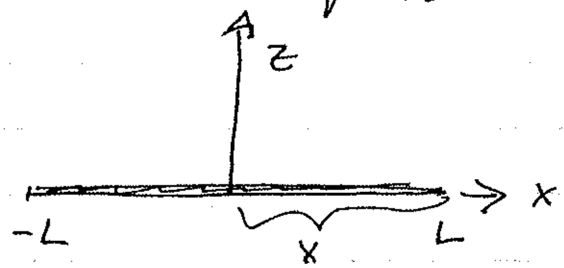
$$\vec{F}_Q = \frac{(-1)}{4\pi\epsilon_0} \frac{qQ}{R^2} \hat{z}$$

* Can verify the answer directly

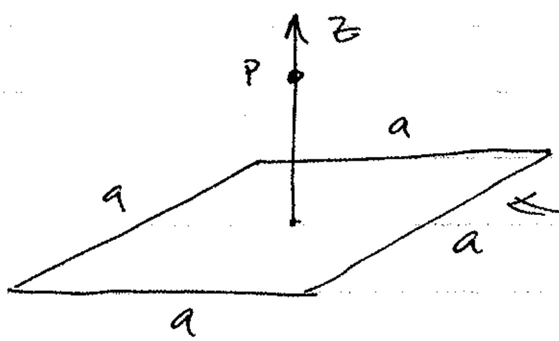
② Prob 2.4

from example 2.1, the field of a charged wire of length $2L$ and line density λ is

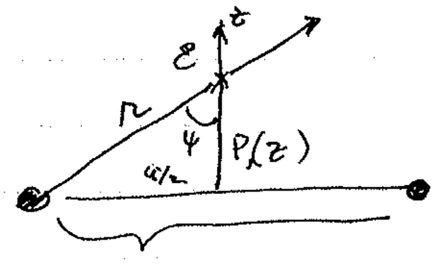
$$E_z = \frac{1}{4\pi\epsilon_0}\,\frac{2\lambda L}{z\sqrt{z^2+L^2}}$$ in the z-direction



Consider a square loop w/ sides a and λ charge per unit length. Find $\vec{E}$ on the z-axis.



← from Ex 2.1, the field due to any side at P will be directed as



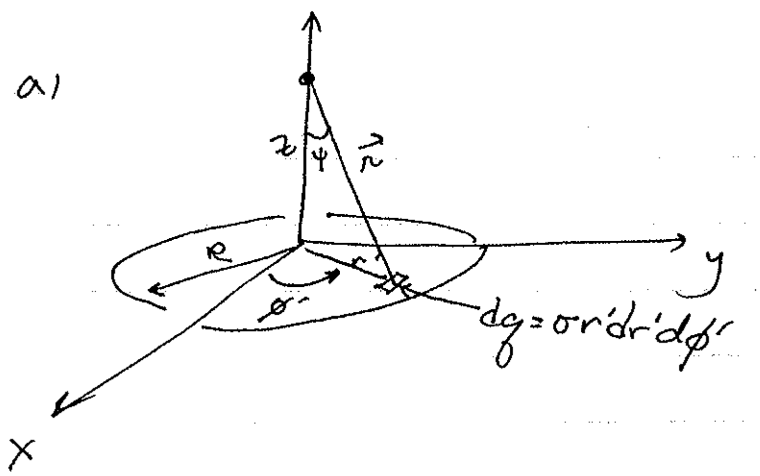
⇒ field projected on z-axis is what we need. So, we want

$$\cos\psi = \frac{z}{\sqrt{\frac{a^2}{4}+z^2}}$$

$$\Rightarrow \mathcal{E}_z = \frac{1}{4\pi\epsilon_0}\,\frac{\lambda a}{z\sqrt{z^2+\frac{a^2}{4}}}\,\frac{z}{\sqrt{z^2+a^2/4}}$$

Total field is $\boxed{4\mathcal{E}_z = \frac{1}{\pi\epsilon_0}\,\frac{\lambda a}{(z^2+a^2/4)}}$

(3) ~~Prob.~~ Prob. 2.6

a) 

By symmetry, we need only calculate the z-compnt for dq

$$\Rightarrow dE_z = \hat{z} \cdot d\vec{E}_q = \frac{\hat{z}\cdot}{4\pi\epsilon_0}\left(\frac{\sigma r' dr' d\phi'}{\mathscr{r}^2}\hat{\mathscr{r}}\right)$$

$$= \frac{\sigma}{4\pi\epsilon_0}\left[\frac{r' dr' d\phi'}{\mathscr{r}^2} \times \frac{z}{\mathscr{r}}\right]$$

$$\Rightarrow E_z = \frac{\sigma z}{4\pi\epsilon_0}\int_0^R dr' \int_0^{2\pi} d\phi' \frac{r'}{(r'^2+z^2)^{3/2}}$$

$$= \frac{\sigma z}{2\epsilon_0}\int_0^R \frac{r' dr'}{(r'^2+z^2)^{3/2}}$$

let $X = r'^2 + z^2 \longrightarrow dX = 2r'dr' \longrightarrow r'dr' = \frac{dX}{2}$

$$= \frac{\sigma z}{2\epsilon_0}\int_{z^2}^{R^2+z^2} \frac{\frac{1}{2}dX}{X^{3/2}}$$

$$= \frac{\sigma z}{4\epsilon_0}\left(\frac{-2}{X^{1/2}}\right)\Bigg|_{z^2}^{R^2+z^2}$$

$$\boxed{E_z = -\frac{\sigma z}{2\epsilon_0}\left[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{|z|}\right]}$$

b) at $x = y = z = 0 \Rightarrow E_z = \frac{\sigma}{2\epsilon_0} \rightarrow F_q = \frac{\sigma q}{2\epsilon_0}$

✓ ✓

c) find E_z for $|h/R| \ll 1$ or $(|z| \ll R)$

$$E_z = -\frac{\sigma z}{2\epsilon_0}\left[\frac{1}{R}\frac{1}{\sqrt{1+z^2/R^2}} - \frac{1}{|z|}\right]$$

$$\approx -\frac{\sigma z}{2\epsilon_0}\left[\frac{1}{R}\left(1-\frac{1}{2}\frac{z^2}{R^2}\right) - \frac{1}{|z|}\right]$$

$$\approx \frac{\sigma}{2\epsilon_0}\left[\frac{z}{|z|} - \left(\frac{z}{R}\right)\right]$$

last question #7

Equation-of-Motion is

$$m\ddot{z} = \frac{\overbrace{q\sigma}^{<0}}{2\epsilon_0}\left[\frac{z}{|z|} - \left(\frac{z}{R}\right)\right] \simeq \frac{q\sigma}{2\epsilon_0}\left[\frac{z}{|z|}\right] \quad *$$

(i) if the charge is released from $z_0 = h$; at rest, $\dot{z}_0 = 0$

$$\Rightarrow \begin{cases} z = \frac{1}{2}\frac{q\sigma}{2\epsilon_0}\overbrace{\left[\frac{z}{|z|}\right]}^{=+1}t^2 + h \\ \dot{z} = \frac{q\sigma}{2\epsilon_0}\left[\frac{z}{|z|}\right]t \end{cases}$$

the charge falls to disk, reaching the disk after time

$$t_f = \sqrt{-\frac{4\epsilon_0 h}{q\sigma}}$$

if a hole is drilled in the disk to allow the charge to pass through the disk, then it moves to a distance $\underline{+h}$ below the disk and stops and then falls back toward the disk, passing through it and returning to height $\underline{h}$. The motion is periodic/period, $P = 4t_f$.

* can also solve "full" problem,

$$m\ddot{z} \simeq \frac{q\sigma}{2\epsilon_0}\left[\frac{z}{|z|} - \frac{z}{R}\right]$$

w/o too much extra effort.

4) Prob. 2.9

Given $\vec{E} = kr^3\hat{r}$, find

a) $\rho(r)$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2E_r = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5) = \rho/\epsilon_0$$

$$5kr^2 = \rho/\epsilon_0$$

$$\boxed{\rho(r) = 5k\epsilon_0 r^2}$$

b) $Q(R)$

(i) $$Q(R) = \int_0^R \rho(r)\, 4\pi r^2 dr$$

$$= 5k\epsilon_0 \int_0^R 4\pi r^4 dr$$

$$\boxed{Q(R) = k\epsilon_0 4\pi R^5}$$

(ii) $$\oint \vec{E}\cdot d\vec{a} = \frac{Q}{\epsilon_0}$$

$$kR^3 4\pi R^2 = \frac{Q}{\epsilon_0}$$

$$4\pi k R^5 = Q/\epsilon_0$$

$$\longrightarrow \boxed{Q = 4\pi k\epsilon_0 R^5}$$

(5) Prob 2.14

Find $\vec{E}$ if $\rho = kr$.

$$\oint_{\vec{r}} \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \int_0^r (kr)\, 4\pi r^2 dr$$

$$E_r 4\pi r^2 = \frac{4\pi k}{\epsilon_0} \frac{r^4}{4}$$

$$\rightarrow \boxed{E_r = \frac{k}{4\epsilon_0} r^2 , \quad r < R = \text{radius of sphere}}$$

⑥ $\vec{E} = \frac{c_0}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$ ← Ideal dipole *

a) Find and sketch the field lines

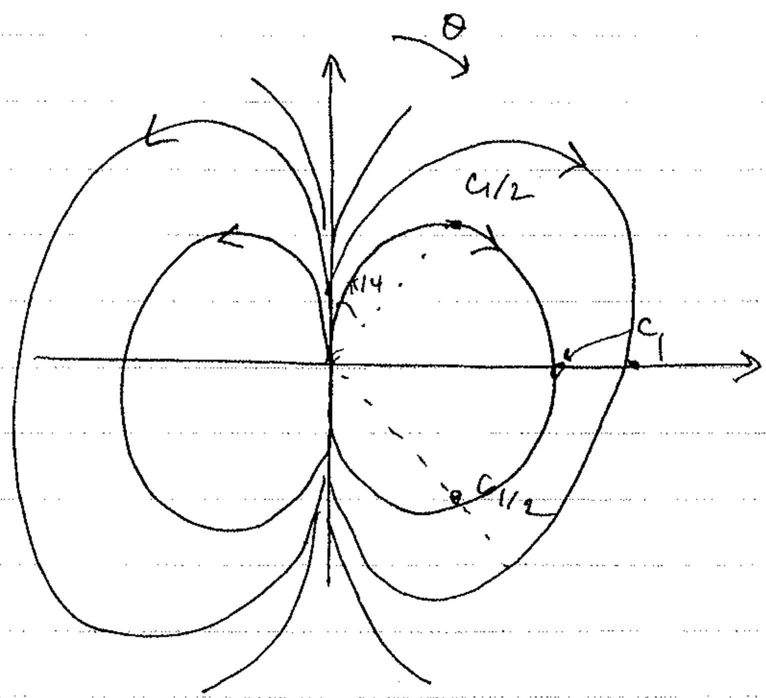
$$\frac{1}{r}\frac{dr}{d\theta} = \frac{E_r}{E_\theta} \longrightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{2\cos\theta}{\sin\theta}$$

$$\text{and } \int \frac{dr}{r} = 2\int \frac{\cos\theta\, d\theta}{\sin\theta}$$

$$\ln r = 2\int \frac{d(+\sin\theta)}{\sin\theta}$$

$$= +2\ln\sin\theta + C$$

$$\boxed{r = C_1 \sin^{+2}\theta}$$; defines a family of sol^ns parameterized by C_1



θ	r
0	0
$\pi/4$	$c_1/2$
$\pi/2$	c_1
$3\pi/4$	$c_1/2$
π	0

* much uglier if a "physical" dipole is used.