Physics 412: Introduction to Electrodynamics
Homework 5
Due: Friday, 2012 November 2

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Problem 2.35

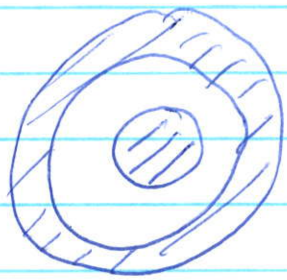
a metal sphere of radius R, carrying charge q is surrounded by a thick concentric shell (inner radius a, outer radius b). The shell carries no net charge.

(a) Find $\sigma(R), \sigma(a), \sigma(b)$

(b) Find $V(0)$ using V_∞ as the reference point

(c) Ground b which lowers its potential to $V=0$. How do your answers to a & b change?

Sol^n



$$\Rightarrow E_r = \begin{cases} 0 \quad , & r<R \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad , & R<r<a \\ 0 \quad , & a<r<b \\ \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad , & b<r \end{cases}$$

if ungrounded

(a) $\sigma(R) = \frac{q}{4\pi R^2}$, $\sigma(a) = -\frac{q}{4\pi a^2}$, $\sigma(b) = \frac{q}{4\pi b^2}$

(b) $-\int_\infty^0 dV = \int_\infty^b \frac{q\,dr}{4\pi\epsilon_0 r^2} + \int_a^R \frac{q\,dr}{4\pi\epsilon_0 r^2} +$ other terms which are 0, because local E_r is 0

$$= \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{b} - \frac{1}{R} + \frac{1}{a}\right]$$

$$\Rightarrow V(0) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right]$$

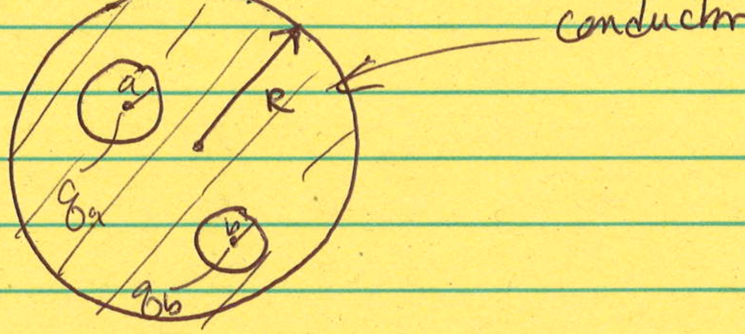
(c) Grad b $\Rightarrow$ $V(b) = 0 \Rightarrow \sigma(b) = 0$
and $\sigma(R)$ and $\sigma(a)$ are unchanged

(c') $-\int_{\infty}^{0} dV = \int_{a}^{R} \frac{q\,dr}{4\pi\epsilon_0 r^2}$ + other terms which are zero because $E_r = 0$

$$= -\frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{a}\right]$$

$$V(0) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{a}\right]$$

(20) Prob 2.36



a) Find σ_a, σ_b, and σ_R

(i) to cancel q_a, spread $-q_a$ onto cavity a

$$\rightarrow \boxed{\sigma_a = -\frac{q_a}{4\pi a^2}}$$

(ii) to cancel q_b, spread $-q_b$ onto cavity b

$$\rightarrow \boxed{\sigma_b = -\frac{q_b}{4\pi b^2}}$$

(iii) for charge neutrality spread (q_a+q_b) onto sphere's surface. To maintain $\vec{E}=0$ in the conductor spread it uniformly

$$\rightarrow \boxed{\sigma_R = \frac{q_a+q_b}{4\pi R^2}}$$

b) what is the field outside of the conductor? a spherical shell of charge (q_a+q_b)

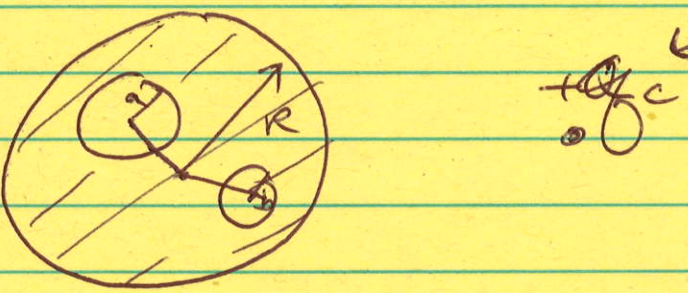
$$\rightarrow \boxed{\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0}\frac{q_a+q_b}{r^2}\hat{r}}$$

c) What is the field in each cavity?

(i) $\vec{E}_a = \frac{1}{4\pi\epsilon_0} \frac{q_a}{r_a^2} \hat{r}_a$, where $\vec{r}_a$ is the vector from the center of the cavity a

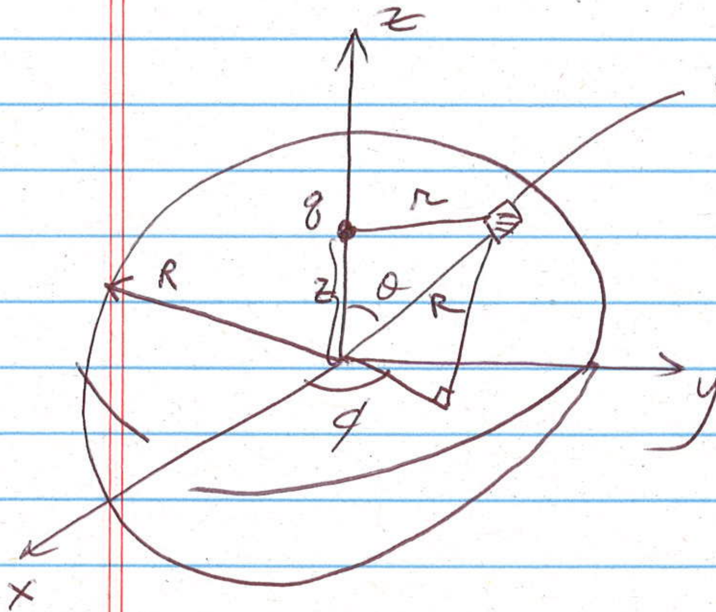
(ii) $\vec{E}_b = \frac{1}{4\pi\epsilon_0} \frac{q_b}{r_b^2} \hat{r}_b$, where $\vec{r}_b$ is the vector from the center of cavity b

d) Consider the addition of charge q_c



Q: What happens? A surface charge is induced at $r = R$ (sphere's surface) which cancels the field of q_c in the conductor.

Problem 3.1



Calulate V on $d\bar{S} = R\sin\theta \, d\phi \, R\, d\theta$ and find the average V on shell

$$\langle V \rangle = \frac{\int \frac{q}{4\pi\epsilon_0} \frac{R^2 \sin\theta \, d\theta \, d\phi}{\sqrt{z^2+R^2-2zR\cos\theta}}}{4\pi R^2}$$

$$= \frac{q}{(4\pi)^2 \epsilon_0} \int \frac{\sin\theta \, d\theta \, d\phi}{\sqrt{z^2+R^2-2zR\cos\theta}}$$

$$= \frac{q}{8\pi\epsilon_0} \int \frac{\sin\theta \, d\theta}{\sqrt{z^2+R^2-2zR\cos\theta}}$$

Let $u = (z^2+R^2-2zR\cos\theta) = du = 2zR\sin\theta \, d\theta$

$$\Rightarrow \langle V \rangle = \frac{q}{8\pi\epsilon_0} \frac{1}{2zR} \int_{(z-R)^2}^{(z+R)^2} \frac{du}{u^{1/2}}$$

$$= \frac{2q}{16\pi\epsilon_0 zR} \Big[u^{1/2} \Big]_{(z-R)^2}^{(z+R)^2}$$

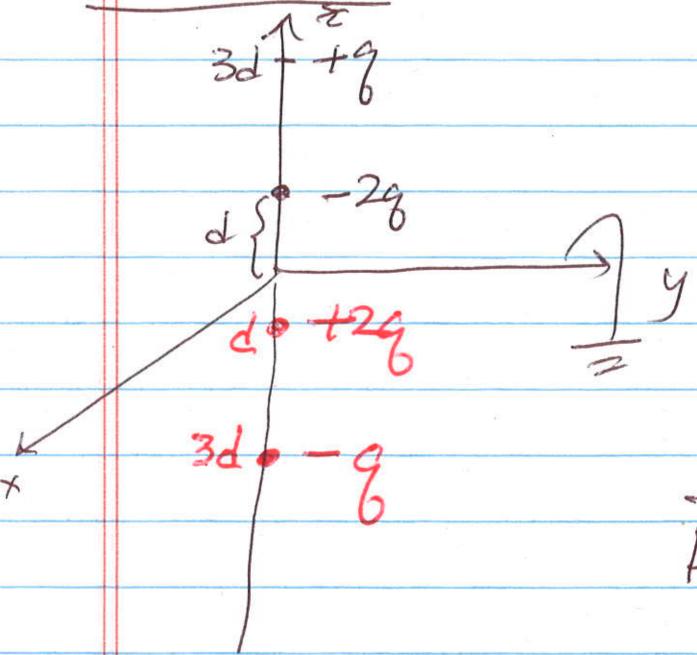
$$= \frac{q}{8\pi\epsilon_0 zR} \Big[(z+R) - (R-z) \Big]$$

$$\boxed{\langle V \rangle = \frac{q}{4\pi\epsilon_0 R}}$$

$$\langle V \rangle = \sum_{i=1}^{N} \frac{q_i}{4\pi\epsilon_0 R} = \frac{Q_{enc}}{4\pi\epsilon_0 R}$$ and by text, we have

$$\langle V \rangle = \frac{Q_{enc}}{4\pi\epsilon_0 R} + V_{center}$$

Problem 3.6



find the force on +q

place 2 image charges as shown (in red).

The force on +q is then

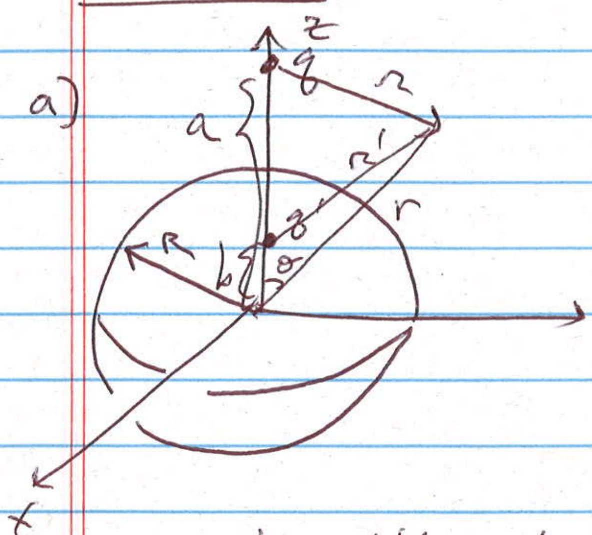
$$\vec{F}_q = \frac{q^2\hat{z}}{4\pi\epsilon_0}\left[-\frac{2}{(2d)^2}+\frac{2}{(4d)^2}-\frac{1}{(6d)^2}\right]$$

$$= \frac{q^2\hat{z}}{4\pi\epsilon_0 d^2}\left[\frac{-29}{72}\right]$$

force pulls q downward

Problem 3.7

a)



$$\left[\begin{array}{l} \mathscr{r}^2 = a^2 + r^2 - 2ar\cos\theta \\ \mathscr{r}'^2 = b^2 + r^2 - 2br\cos\theta \end{array}\right.$$

from figure

$$\Rightarrow V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{a^2+r^2-2ar\cos\theta}} + \frac{q'}{\sqrt{b^2+r^2-2br\cos\theta}}\right]$$

okay, let's get rid of q' & b

$$q' = -\frac{R}{a}q, \quad b = R\left(\frac{R}{a}\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{a^2+r^2-2ar\cos\theta}} - \frac{\left(\frac{R}{a}\right)q}{\sqrt{R^2\left(\frac{R}{a}\right)^2 + r^2 - 2R^2\left(\frac{R}{a}\right)r\cos\theta}}\right]$$

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{a^2+r^2-2ar\cos\theta}} - \frac{q}{\sqrt{R^2 + r^2\left(\frac{a}{R}\right)^2 - 2ar\cos\theta}}\right]}$$

b) Find σ: get $\vec{E}\cdot\hat{r}$ at $r=R$

$$E_r = \frac{-q}{4\pi\epsilon_0}\left[\frac{-\frac{1}{2}(2r-2a\cos\theta)}{(a^2+r^2-2ar\cos\theta)^{3/2}} - \frac{-\frac{1}{2}\{2r\left(\frac{a}{R}\right)^2 - 2a\cos\theta\}}{\left(R^2+r^2\left(\frac{a}{R}\right)^2-2ar\cos\theta\right)^{3/2}}\right]$$

at $= r = R$

$$E_r(r=R) = \frac{-q}{4\pi\epsilon_0}\left[\frac{-(R - a\cos\theta)}{(a^2+R^2-2aR\cos\theta)^{3/2}} + \frac{a^2/R - a\cos\theta}{(R^2+a^2-2aR\cos\theta)^{3/2}}\right]$$