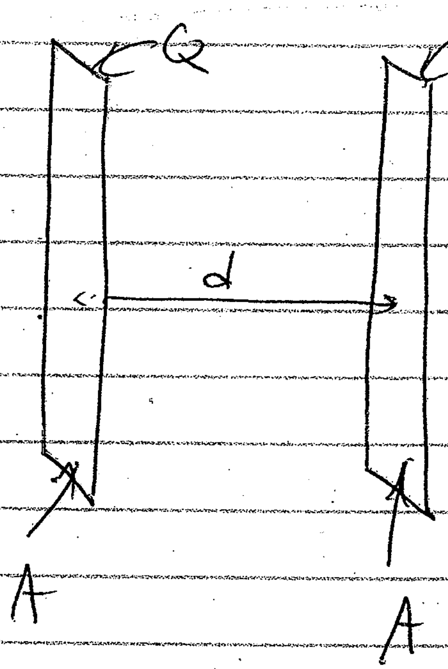
Prob. 2.37



Let $A \gg d^2$. Find the electrostatic pressure on each plate.

Sol^n

a) Because $A \gg d^2 \Rightarrow \sigma = \frac{Q}{A}$

$$\rightarrow \vec{f} = \frac{1}{2\epsilon_0}\sigma^2\hat{n} = \frac{1}{2\epsilon_0}\frac{Q^2}{A^2}\hat{n}$$

force per unit area and the pressure is (by def^n)

$$\boxed{P = \frac{1}{2\epsilon_0}\left(\frac{Q}{A}\right)^2}$$

Prob 2.43

Find the net force that the southern hemisphere of a uniformly charged sphere exerts on the northern hemisphere. (Answer: $\frac{1}{4\pi\epsilon_0}\frac{3Q^2}{16R^2}$)

Sol^{n}

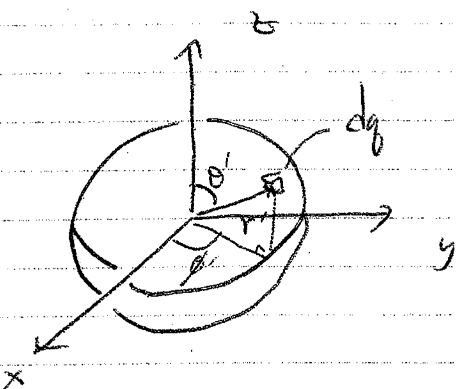
ⓐ for a uniformly charged sphere (for $r<R$)

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q(r)}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{\cancel{3}Q}{\cancel{4\pi}R^3}\right)r^3\frac{\hat{r}}{r^2}\left(\frac{\cancel{4\pi}}{\cancel{3}}\right)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{\overset{\rho}{\cancel{3}Q}}{\cancel{4\pi}R^3}\right)r\hat{r}$$

b) $d\vec{F} = \rho d^3x'\vec{E}(\vec{r}')$



By symmetry we need only keep F_z component,

$$dF_z = d\vec{F}\cdot\hat{z} = \rho d^3x'\vec{E}(\vec{r}')\cdot\hat{z}$$

$$= \rho d^3x'\frac{1}{4\pi\epsilon_0}\left(\frac{\cancel{3}Q}{\cancel{4\pi}R^3}\right)r'\underbrace{\hat{r}'\cdot\hat{z}}_{\cos\theta'}$$

$$F_z = \frac{\cancel{3}\rho Q}{\cancel{4}\pi\epsilon_0 R^3}\int_0^{2\pi}d\phi'\int_0^R dr'\int_0^{\frac{\pi}{2}}r'^3\sin\theta' d\theta'\cos\theta'$$

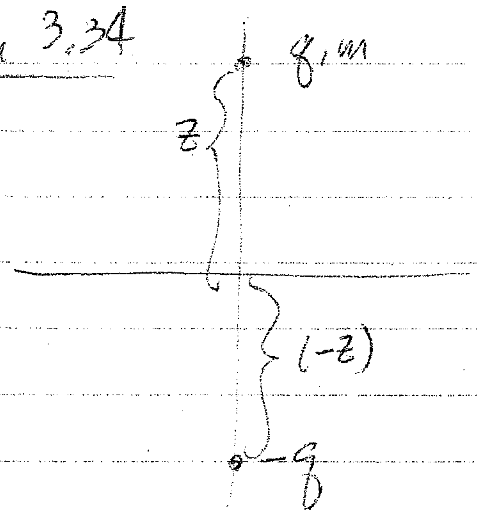
$$= \frac{\cancel{3}\left(\frac{3Q}{4\pi R^3}\right)Q}{\cancel{4}\pi\epsilon_0 R^3}2\pi\left(\frac{R^4}{4}\right)\int_0^{\frac{\pi}{2}}\sin\theta'\cos\theta' d\theta'$$

$$F_z = \frac{\left(\frac{3Q}{4\pi R^3}\right)Q}{4\pi\epsilon_0 R^3} 2\pi\left(\frac{R^4}{4}\right)\int_0^{\frac{\pi}{2}} \sin\theta' \cos\theta' d\theta'$$

$$= \frac{3Q^2}{4\pi\epsilon_0 \times 8R^2}\left[\frac{\sin^2\theta'}{2}\Big|_0^{\frac{\pi}{2}}\right]$$

$$\boxed{F_z = \frac{3Q^2}{4\pi\epsilon_0 16R^2}}$$

Problem 3.34



$$\Rightarrow m\ddot{z} = q\left(\frac{-q}{4\pi\epsilon_0 (2z)^2}\right)$$

$$\boxed{\ddot{z} = -\frac{q^2}{16\pi\epsilon_0 m z^2}}$$

Equation-of-motion

Solve

let $P = \dot{z} \Rightarrow \ddot{z} = \frac{d}{dt}P = \frac{dz}{dt}\frac{dP}{dz} = P\frac{dP}{dz} = \frac{d}{dz}\left(\frac{P^2}{2}\right)$

and so,

$$\frac{d}{dz}\left(\frac{P^2}{2}\right) = -\frac{q^2}{16\pi\epsilon_0 m z^2}$$

$$\Rightarrow \frac{P^2}{2} = +\underbrace{\frac{q^2}{16\pi\epsilon_0 m}}\left(\frac{1}{z}\right)\Bigg|_d^z$$

start from rest

$$= \alpha\left(\frac{1}{z} - \frac{1}{d}\right)$$

$$\Rightarrow \frac{dz}{dt} = \sqrt{2\alpha}\sqrt{\frac{1}{z} - \frac{1}{d}} = \sqrt{2\alpha}\sqrt{\frac{d-z}{dz}}$$

$$\Rightarrow \int\left(\sqrt{\frac{d-z}{zd}}\right)^{-1} dz = \int \sqrt{2\alpha}\, dt = \sqrt{2\alpha}\, t$$

$$\Rightarrow \sqrt{2\alpha}\, t = \sqrt{d}\left[-\sqrt{z(d-z)} + \frac{d}{2}\left\{2\tan^{-1}\sqrt{\frac{d-z}{z}}\right\}\right]_d^0$$

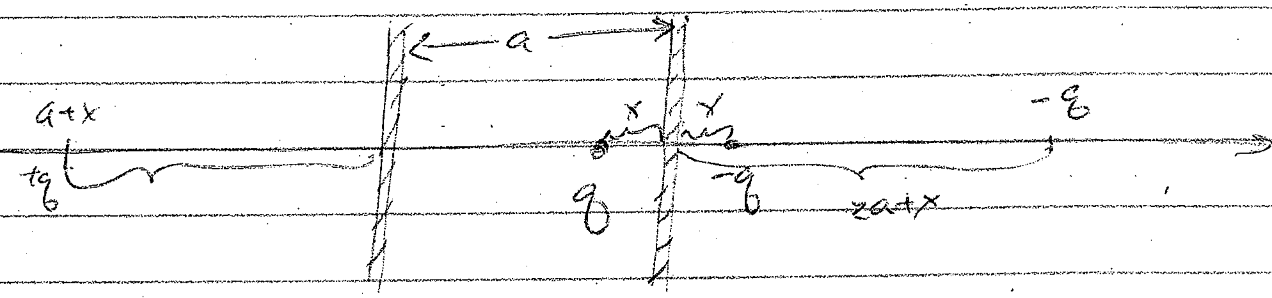
$$\rightarrow t = d\sqrt{\frac{d}{2\alpha}}\,\frac{\pi}{2}$$

$$= \frac{\pi d}{2}\sqrt{\frac{d}{2\left[\frac{q^2}{16\pi\epsilon_0 m}\right]}}$$

$$= \frac{\pi d}{2}\sqrt{\frac{8\pi\epsilon_0 m d}{q^2}}$$

$$\boxed{t = \frac{(\pi d)^{3/2}}{q}\sqrt{2\pi\epsilon_0 m}}$$

Problem 3.35



a) Consider the series formed by first considering the right hand plate. The fields points to the right,

$$\Rightarrow E_R = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(2x)^2} + \frac{1}{(2a)^2} + \frac{1}{(2a+2x)^2} + \frac{1}{(4a)^2} + \frac{1}{(4a+2x)^2} + \cdots\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\sum_{i=1}^{\infty}\frac{1}{(2i)^2a^2} + \sum_{i=1}^{\infty}\frac{1}{[2(i-1)a+2x]^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\sum_{i=1}^{\infty}\left\{\frac{1}{(2ia)^2} + \frac{1}{2^2([i-1]a+x)^2}\right\}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{2a}\right)^2\sum_{i=1}^{\infty}\left\{\frac{1}{i^2} + \frac{1}{([i-1]+\frac{x}{a})^2}\right\}$$

b) Now, look at the series formed by first considering the left plate. The field points to the left,

$$\Rightarrow E_L = -\frac{q}{4\pi\epsilon_0}\left[\frac{1}{2^2(a-x)^2} + \frac{1}{2^2(a)^2} + \frac{1}{[4a-2x]^2} + \frac{1}{(4a)^2} + \frac{1}{\quad} + \cdots\right]$$

$$\bar{E} = -\frac{q}{4\pi\epsilon_0}\left[\frac{1}{2^2}\frac{1}{(a-x)^2} + \frac{1}{2^2}\frac{1}{a^2} + \frac{1}{2^2}\frac{1}{(2a-x)^2} + \frac{1}{2^2}\frac{1}{(2a)^2} + \frac{1}{2^2}\frac{1}{(3a-x)^2} + \cdots\right]$$

$$= -\frac{q}{4\pi\epsilon_0}\left(\frac{1}{2a}\right)^2\left[\frac{1}{(1-\frac{x}{a})^2} + 1 + \frac{1}{(2-\frac{x}{a})^2} + \frac{1}{2^2} + \frac{1}{(3-\frac{x}{a})^2} + \cdots\right]$$

$$= -\frac{q}{4\pi\epsilon_0}\left(\frac{1}{2a}\right)^2\sum_{i=1}^{\infty}\left[\frac{1}{i^2} + \frac{1}{(i-\frac{x}{a})^2}\right]$$

So, the force on q is, $F = q(E_R + E_L)$, that is,

$$F = q\left(\frac{q}{4\pi\epsilon_0}\right)\left(\frac{1}{2a}\right)^2\sum_{i=1}^{\infty}\left\{\frac{1}{i^2} + \frac{1}{(i-1+\frac{x}{a})^2} - \frac{1}{i^2} - \frac{1}{(i-\frac{x}{a})^2}\right\}$$

$$\boxed{F = \frac{q^2}{16\pi\epsilon_0 a^2}\sum_{i=1}^{\infty}\left\{\frac{1}{(i-1+\frac{x}{a})^2} - \frac{1}{(i-\frac{x}{a})^2}\right\}}$$

Special places

a) $x = a/2$

$$F = \frac{q^2}{16\pi\epsilon_0 a^2}\sum_{i=1}^{\infty}\left[\frac{1}{(i-\frac{1}{2})^2} - \frac{1}{(1-\frac{1}{2})^2}\right]$$

$= 0$ ✓

b) $x = 0$

$$F = \frac{q^2}{16\pi\epsilon_0 a^2}\sum_{i=1}^{\infty}\left[\frac{1}{(i-1)^2} - \frac{1}{i^2}\right] \longrightarrow \infty \checkmark$$

c) $a \to \infty$ (depends on x/a)

$$F = \frac{q^2}{16\pi\epsilon_0}\sum_{i=1}^{\infty}\left\{\frac{1}{[(i-1)a+x]^2} - \frac{1}{[ia-x]^2}\right\}$$

each i term, F_i,

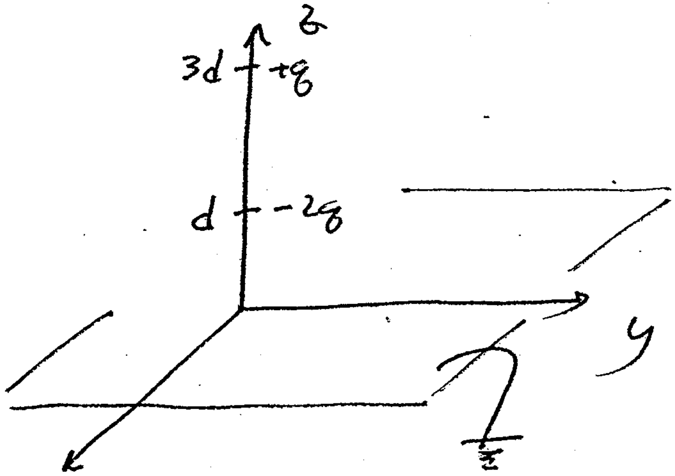
(i) if $a \to \infty$, $F_i \to 0$ for $i \neq 1$

(ii) $i = 1 \Rightarrow F_1 = \frac{q^2}{16\pi\epsilon_0}\left[\frac{1}{x^2} - \frac{1}{(a-x)^2}\right]$

$$\approx \frac{q^2}{16\pi\epsilon_0 x^2}, \quad a \to \infty$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2x)^2} \checkmark$$

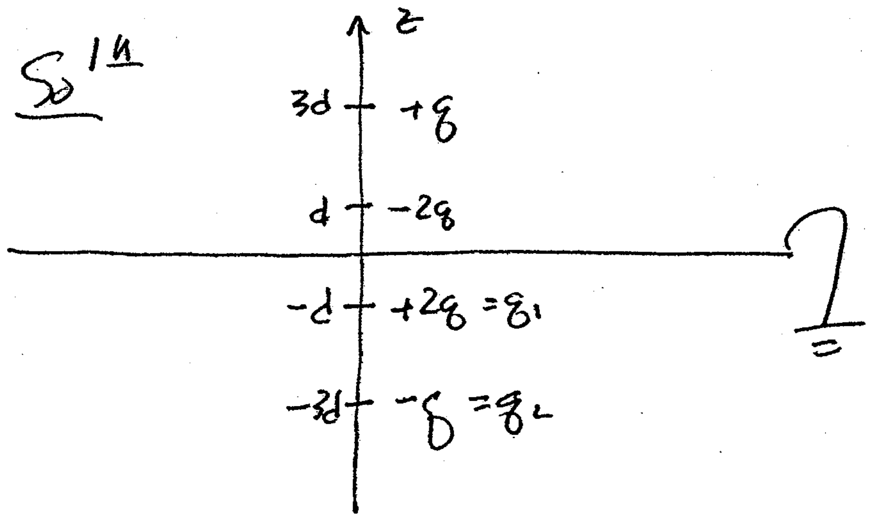
Prob. 3.6



Conducting planar surface in xy plane. The conductor is grounded.

Find the force on charge +q

Soln



To force $V(z=0)=0$, we need 2 images as shown above. The force on +q is then

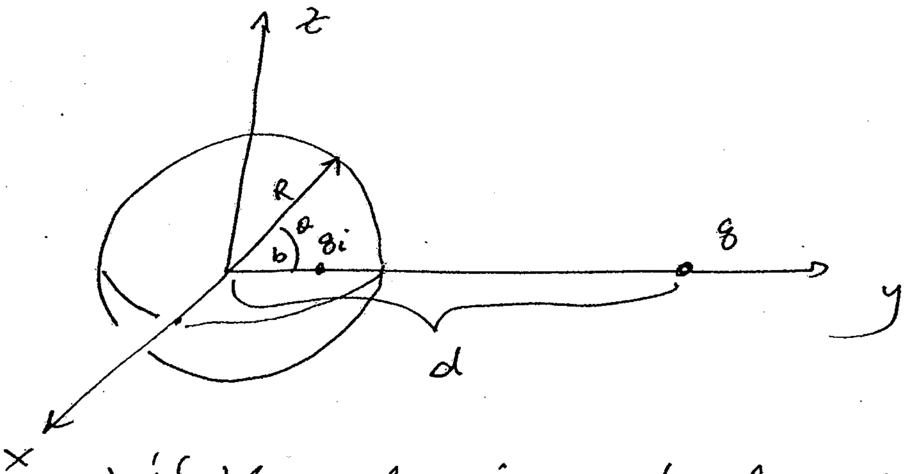
$$\vec{F}_z = \frac{q\hat{z}}{4\pi\epsilon_0}\left[-\frac{2q}{(3d-d)^2}+\frac{2q}{(3d+d)^2}-\frac{q}{(3d+3d)^2}\right]$$

$$= \frac{q^2\hat{z}}{4\pi\epsilon_0}\left[-\frac{1}{2d^2}+\frac{1}{8d^2}-\frac{1}{36d^2}\right]$$

$$= \frac{q^2\hat{z}}{4\pi\epsilon_0 d^2}\left[\frac{-36+9-2}{72}\right]$$

$$= -\frac{q^2}{\ldots}\,\hat{z}\left(\frac{29}{\ldots}\right)$$

Prob. 3.8



a) if the sphere is insulated and held at $V = V_0$

$\Rightarrow$ place charge at center of sphere. Charge should have size $\ni$

$$V_0 = \frac{q_i}{4\pi\epsilon_0 R} \longrightarrow \boxed{q_i = 4\pi\epsilon_0 R V_0}$$

b) what is the force of attraction between a neutral conducting sphere and a point charge q.

(i) $q_i = -q\left(R/d\right)$ for $V = 0$ (grounded sphere)

$\Rightarrow$ we need to add $q_i' = q\left(R/d\right)$ at the center to make $Q_c = 0$.

$$\Rightarrow \vec{F}_q = \frac{q\hat{y}}{4\pi\epsilon_0}\left[-\frac{q R/d}{(d - R^2/d)^2} + \frac{q R/d}{d^2}\right]$$

$$= \frac{q^2 (R/d)}{4\pi\epsilon_0 d^2}\hat{y}\left[1 - \frac{1}{(1 - R^2/d^2)^2}\right]$$

$$= \frac{q^2 (R/d)\hat{y}}{4\pi\epsilon_0 d^2}\left[\frac{-2R^2/d^2 + R^4/d^4}{(1 - R^2/d^2)^2}\right]$$

$$= \frac{q^2 \left(R/d\right)^5 \hat{y}}{\ldots}\left[\frac{1 - 2(d^2/R^2)}{\ldots}\right]$$