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Physics 412: Introduction to Electrodynamics
Test 1
Monday, 2012 October 22

Do Problems 1 and 2, and then either Problem 3 or 4.

Problem 1:

a. Find the electric field $\mathbf{E}$ on axis for a cylindrical shell of charge. The cylindrical shell has radius R, length $2R$, and uniform surface charge density σ. Also, consider only the sides of the cylinder when calculating the field; the endcaps of the cylinder carry no charge. (8 points)

b. Find an expression for the electric field $\mathbf{E}$ valid in a small region around the midpoint of the cylinder on the symmetry axis. (8 points)

c. A particle of mass m and charge q placed at the midpoint of the cylinder on its symmetry axis, feels no force. If $q\sigma < 0$ and the particle is displaced slightly from equilibrium by an amount δh and then released, it will execute periodic motion about equilibrium. What is the particle's frequency of oscillation? Ignore gravity for this question. (2 points)

Problem 2:

a. An infinite cylinder has uniform charge density $\rho_\circ$ and radius R_c. Find its electric field **E**. (8 points)

b. A cavity is hollowed out of the cylinder in part a. The axis of the cavity is parallel to the axis of the cylinder. The axes of the cylinder and cavity are separated by distance a. The radius of the cavity is R_b. The dimensions of the cylinder and cavity obey the inequalities, $a > R_b$ and $R_b + a < R_c$. Find the electric field **E** in the cavity. (*Hint:* Use superposition to make a zero charge cavity.) (8 points)

Problem 3:

Four equal charges, q, are placed at the corners of a square. The sides of the square have length l. A charge Q is placed at the center of the square.

a. Find the charge Q so that all charges, including Q, feel zero force. (8 points)

b. Find the energy, W, of the charge configuration. (6 points)

c. Is the configuration bound or unbound? Is the configuration stable? Justify your answer (2 points)

Problem 4:

For the electric field:

$$E = \begin{cases} \frac{Q}{4\pi\epsilon_\circ}\frac{\hat{r}}{r^2} & r > R \\ \frac{Q}{4\pi\epsilon_\circ}\frac{\hat{r}}{R^2} & r \leq R \end{cases} \tag{1}$$

a. find the charge density $\rho(r)$ (4 points)

b. find the scalar potential V(r) (6 points)

c. find the energy contained in the fields (6 points)