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Physics 412: Introduction to Electrodynamics

Test 2

Monday, 2012 November 19

Do problems 1 AND 2, and problem 3 OR 4. Mark clearly the problem you wish to have considered.

Problem 1 (16 points):

A spherical shell of radius R has potential

$$V(R, \theta) = V_0 \cos^3 \theta \quad (1)$$

- a. Express the potential at $r = R$ as a linear combination of Legendre polynomials. (4 points)
- b. Find the potential inside and outside the shell for the boundary condition given above and $V = 0$ at ∞ . (12 points)

Problem 2 (16 points):

A spherical shell of radius R carries uniform surface charge density $+\sigma_0$ on its *northern hemisphere* and a uniform surface charge density $-\sigma_0$ on its *southern hemisphere*.

- a. Find the potential on the z -axis, where the z -axis is the symmetry axis of the sphere. (6 points)
- b. Find an expression for the potential, valid away from the z -axis, for $r \gg R$. Keep the two lowest order nonzero multipole fields. (10 points)

Problem 3 (18 points):

A large conducting sphere contains a spherical cavity of radius b . The spherical conductor is grounded and large in the sense that its radius R is $\gg b$.

- a. A point charge Q is placed at the center of the cavity. Find the potential in the cavity and the surface charge density σ induced on the wall of the cavity. (6 points)
- b. The point charge Q is removed from the cavity and an ideal dipole with moment \mathbf{p} placed at the center of the cavity. Find the potential in the interior of the cavity and the surface charge density σ induced on the wall of the cavity. (12 points)

Problem 4 (18 points):

A hemispherical boss with radius a protrudes from the surface of a semi-infinite slab. The slab and boss are made of conducting material and are grounded. The z -axis passes through the center of the hemispherical boss and is normal to the semi-infinite slab. A charge q is placed on the z -axis at $z = h$ where $h > a$.

- a. Using the image charge method, find the potential $V(\mathbf{r})$ outside the boss and slab. (10 points)
- b. Find the force on charge q . Does the hemispherical boss increase the force between the charge q and the conductor compared to the situation without a boss, that is, compared to the case of a point charge q a distance h above a grounded semi-infinite planar conductor? (5 points)
- c. Find the total charge induced on the conductor. (3 points)