c) apply BCs

(i) at $r \to \infty$, $W \to 0$

$$\Rightarrow V_m^> = \sum_{l=0}^{\infty}\left(\cancelto{0}{A_l^>}\, r^l + \frac{B_l^>}{r^{l+1}}\right) P_l(\cos\theta)$$

at $r \Rightarrow 0$, W must be regular

$$\Rightarrow V_m^< = \sum_{l=0}^{\infty}\left(A_l^< r^l + \cancelto{0}{\frac{B_l^<}{r^{l+1}}}\right) P_l(\cos\theta)$$

(ii) at $r = R$, V_m is continuous

$$\Longrightarrow \frac{B_l^>}{R^{l+1}} = A_l^< R^l$$

(iii) at $r = R$, $\vec{\nabla}\cdot\vec{B} = 0 = \vec{\nabla}\cdot\left[\mu_0 \vec{H} - \mu_0 \vec{M}\right]$

$$\Rightarrow \vec{\nabla}\cdot\vec{H} = \vec{\nabla}\cdot\vec{M}$$ apply ~~Gauss's law~~ divergence theorem

$$\oint \vec{H}\cdot d\vec{S} = \int (\vec{\nabla}\cdot\vec{M})\, d^3x$$

$$\Rightarrow H_r^> \cdot A - H_r^< \cdot A = M_0 A \cos\theta$$

$$-\frac{\partial V_m^>}{\partial r} - \left(-\frac{\partial V_m^<}{\partial r}\right) = M_0 \cos\theta \qquad l = 1$$

$$-\left[\underset{l=1}{-(l+1)}\frac{B_1^>}{R^{l+2}}\right] + \underset{l=1}{l A_l^<} R^{l-1} = M_0$$

$$2\frac{B_1^>}{R^3} + A_1^< = M_0$$

$$(ii) \ \& \ (iii) \Longrightarrow \frac{2}{R^3}\left[A_1^< R^3\right] + A_1^< = M_0$$

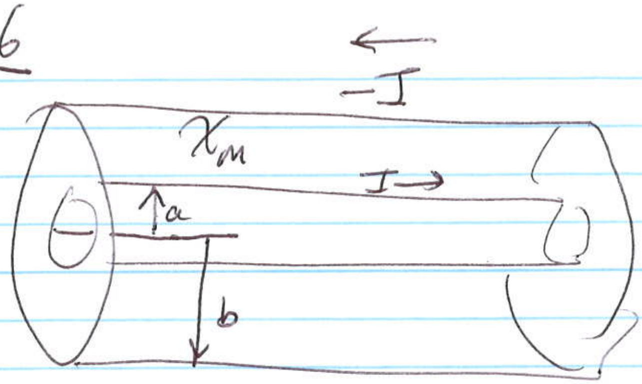
$$\boxed{A_1^< = \frac{M_0}{3}}$$

ad

$$\boxed{B_1^> = \frac{M_0}{3} R^3}$$

$$\Rightarrow V_m = \begin{cases} \frac{M_0 r}{3} \cos\theta & , r < R \\ \frac{M_0 R^3}{3 r^2} \cos\theta & , r > R \end{cases}$$

6.16



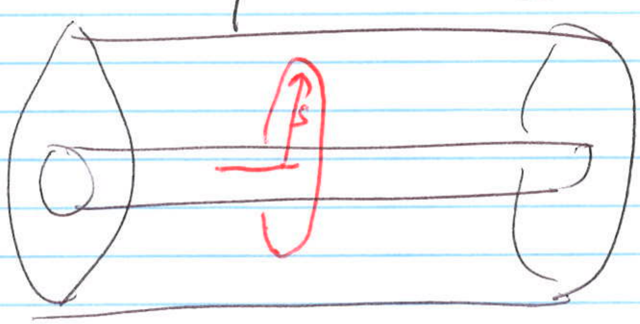
find $\vec{B}$ for $a < r < b$

(a) Use Ampere's law $\oint \vec{B}\cdot d\vec{l} = \mu_0 \int (\vec{J}_f + \vec{J}_b)\cdot d\vec{S}$ directly or use

$$\vec{\nabla}\times\vec{H} = \vec{J}_f \Rightarrow \oint \vec{H}\cdot d\vec{l} = \int \vec{J}_f \cdot d\vec{S}$$

the latter is clearly preferred. Again the field will be azimuthal

$\Rightarrow H_\phi 2\pi s = I$ if we use the loop



$$\Rightarrow \boxed{\vec{H}_\phi = \frac{I}{2\pi s}\hat{\phi}}$$

in desired region

take the material as linear

$\Rightarrow \vec{B} = \mu\vec{H}$ and $\boxed{\vec{B} = \frac{\mu I}{2\pi s}\hat{\phi}}$

where $\mu = \mu_0(1+\chi_m)$

(b) find magnetization

$$\vec{B} = \mu_0(\vec{H}+\vec{M}) \Rightarrow \vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$

$$= \frac{\mu}{\mu_0}\frac{I}{2\pi s}\hat{\phi} - \frac{I}{2\pi s}\hat{\phi}$$

$$= \frac{I}{2\pi s}\hat{\phi}\left(\frac{\mu}{\mu_0}-1\right)$$

$$= \frac{I}{2\pi s}\hat{\phi}\left(\frac{\mu_0(1+\chi_m)}{\mu_0}-1\right)$$

$$\boxed{\vec{M} = \chi_m \frac{I}{2\pi s}\hat{\phi} = \chi_m \vec{H}}$$

(c) find bound currents

(i) $$\vec{J}_b = \vec{\nabla}\times\vec{M} = \vec{\nabla}\times\left(\frac{\chi_m I}{2\pi}\right)\frac{\hat{\phi}}{s}$$

$$= \frac{\chi_m I}{2\pi}\left[-\hat{s}\left(\frac{\partial}{\partial z}M_\phi\right) + 0 + \frac{\hat{z}}{s}\left(\frac{\partial}{\partial s}sM_\phi\right)\right] = 0$$

(ii) $$\vec{K}_b = \vec{M}\times\hat{n} = \begin{cases} \frac{\chi_m I}{2\pi a}\hat{z} & , s=a \\ \frac{\chi_m I}{2\pi b}(-\hat{z}) & , s=b \end{cases}$$

(iii) $\vec{K}_b$s correspond to

at $s=a$ $I_b = \oint|\vec{K}_b|\,dl = \frac{\chi_m I}{2\pi a}2\pi a = \chi_m I$

at $s=b$ $I_b = -\chi_m I$

(d) total currents are then

at $s=a$, $I_a = I + \chi_m I = (1+\chi_m) I$

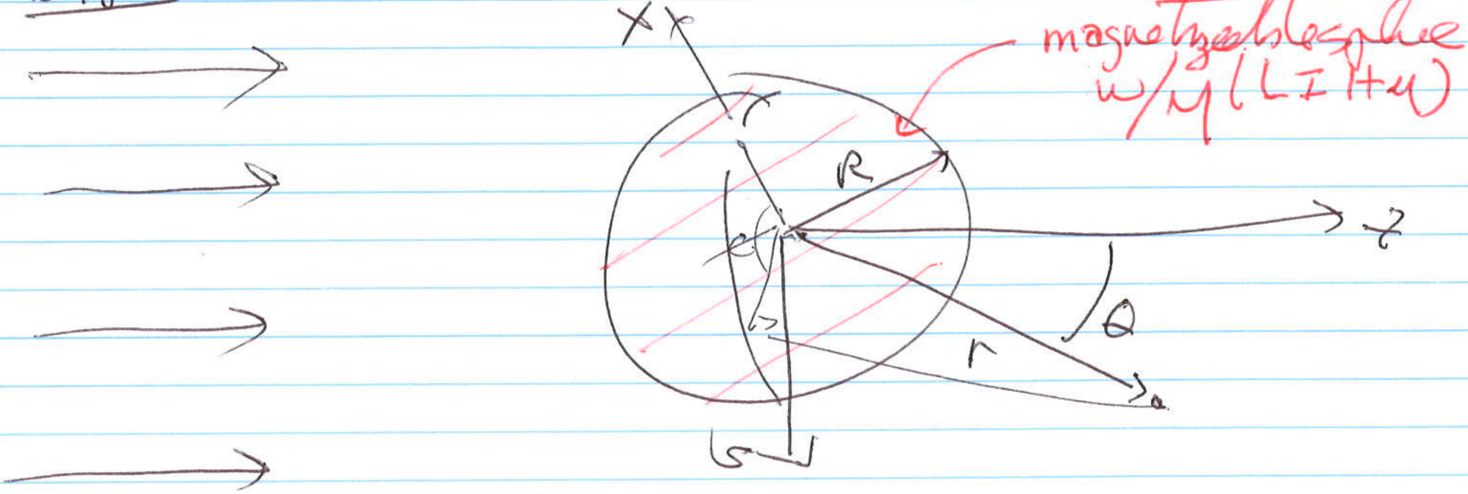
at $s=b$, $I_b = -I - \chi_m I = -(1+\chi_m) I$

e) Recall in $a < s < b$,

$$\vec{B} = \frac{\mu I}{2\pi s}\hat{\phi} = \frac{\mu_0 (1+\chi_m) I}{2\pi s}\hat{\phi}$$

in agreement w/ (d)

6.18



(i) at ∞, $\vec{B} = B_0 \hat{z} \rightarrow \vec{H} = \frac{B_0}{\mu_0}\hat{z} \Rightarrow V_m = -\frac{B_0}{\mu_0} z = -\frac{B_0}{\mu_0} r\cos\theta$

find $\vec{B}$ everywhere

a) again we'll have $\nabla^2 V_m = 0$ everywhere except at $r=R$

$$\Rightarrow V_m = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

b) at $r \rightarrow \infty$, $V_m \rightarrow -\frac{B_0}{\mu_0} r\cos\theta \Rightarrow l = 1$ and so

$$V_m^{>} = \left(A_1^{>} r + \frac{B_1^{>}}{r^2}\right)\cos\theta$$

c) at $r = 0$, V_m must be regular $\Rightarrow B_l^{<} = 0$ and

$$V_m^{<} = A_1^{<} r\cos\theta$$

d) V_m is continuous at $r=R$

$$\Rightarrow A_1^> R + \frac{B_1^>}{R^2} = A_1^< R$$

and $\boxed{B_1^> = A_1^< R^3 - A_1^> R^3}$

e) $\vec{\nabla} \cdot \vec{B} = 0$ at $r=R$

$\rightarrow$ ~~$\vec{B}$~~ ~~&~~ ΔB_r is continuous

$$\Rightarrow B_r^> = B_r^<$$

$$\Rightarrow \mu_0 H_r^> = \mu H_r^<$$

$$-\mu_0 \frac{\partial V_m^>}{\partial r} = -\mu \frac{\partial V_m^<}{\partial r}$$

$$\mu_0 \left[A_1^> - 2\frac{B_1^>}{R^3} \right] = \mu \left[A_1^< \right]$$

$$\boxed{B_1^> = \frac{R^3}{2\mu_0} \left[\mu_0 A_1^> - \mu A_1^< \right]}$$

$$\Rightarrow A_1^< R^3 - A_1^> R^3 = A_1^> \frac{R^3}{2} - \frac{\mu R^3}{2\mu_0} A_1^<$$

$$R^3 A_1^< \left[1 + \frac{\mu}{2\mu_0} \right] = \frac{3}{2} R^3 A_1^> = -\frac{3}{2} R^3 \frac{B_0}{\mu_0}$$

$$A_1^> = -\frac{B_0}{\mu_0}$$ from BC at ∞

$$\Rightarrow A_1^{<} = -\frac{\frac{3}{2}\frac{B_0}{\mu_0}}{1+\frac{\mu}{2\mu_0}} = -\frac{3}{2}\frac{B_0}{\left(\frac{\mu}{2}+\mu_0\right)}$$

ad

$$B_1^{>} = \frac{R^3}{2\mu_0}\left[\cancel{\mu_0}\left(-\frac{B_0}{\cancel{\mu_0}}\right) - \mu\,\frac{3}{2}\,\frac{B_0}{\frac{\mu}{2}+\mu_0}\right]$$

$$= -\frac{B_0 R^3}{2\mu_0}\left[1+\frac{3}{2}\frac{\mu}{\frac{\mu}{2}+\mu_0}\right]$$

$$= -\frac{B_0 R^3}{2\mu_0}\left[\frac{5\mu+2\mu_0}{2\left(\frac{\mu}{2}+\mu_0\right)}\right]$$