Homework 6

Due: February 18, 2013

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26. 6.9

27. 6.12

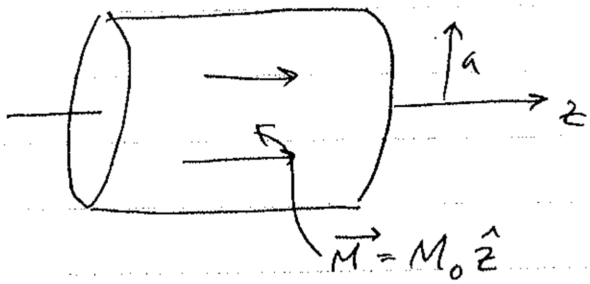
28. 6.16

29. 6.18

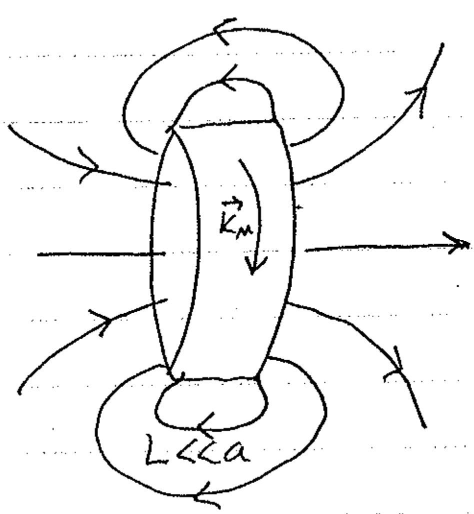
30. 6.20

31. 6.26

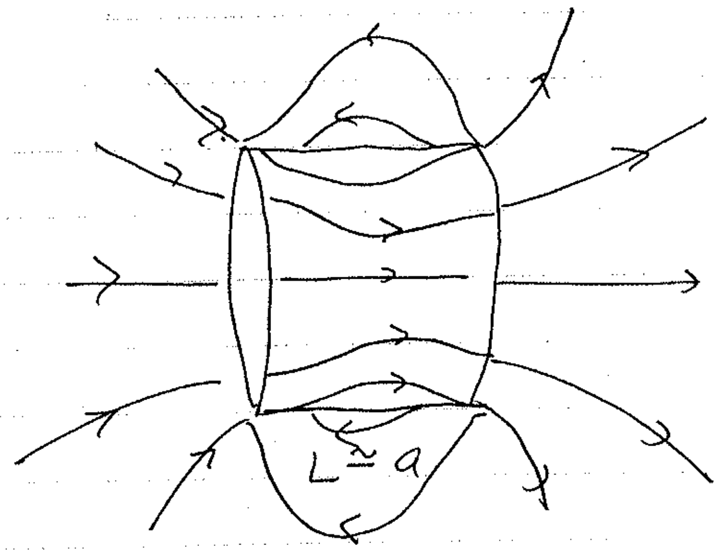
29. Problem 6.9



a) $\vec{J}_M = \vec{\nabla} \times \vec{M} = 0$, $\vec{K}_M = \vec{M} \times \hat{S} = M_0 \hat{\phi}$

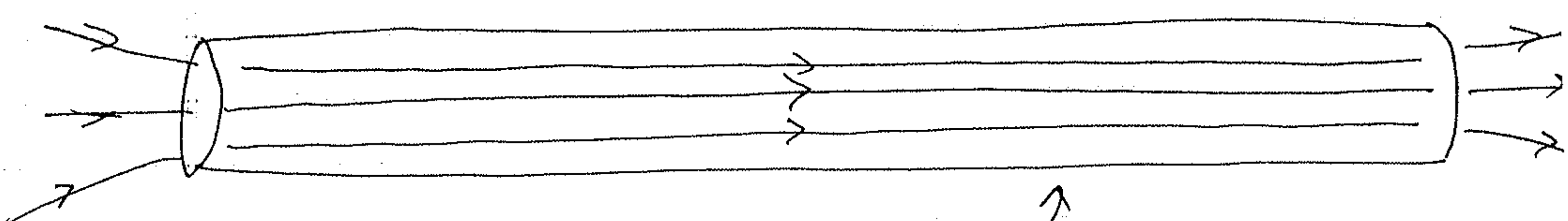


resembles a wire loop



from class notes

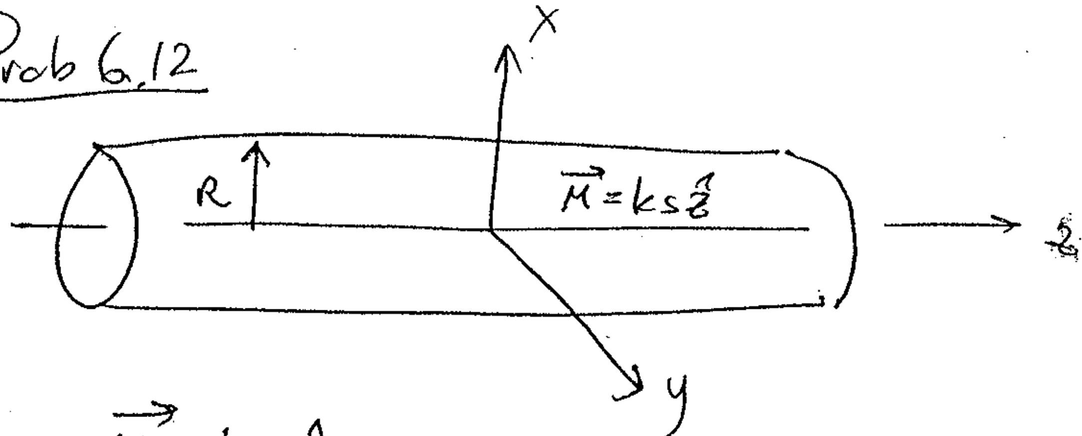
weak external $\vec{B}$



resembles an infinite solenoid

$L \gg a$

Prob 6.12



$\vec{M} = ks\hat{z} \longrightarrow$

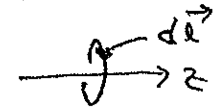
(i) $\vec{J}_M = \vec{\nabla} \times \vec{M}$
$= -k\hat{\phi}$

(ii) $\vec{K}_M = \vec{M} \times \hat{s}$ at $s=R$
$= kR\hat{\phi}$

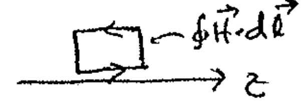
Find $\vec{B}$ using 2 methods

b) $\vec{\nabla} \times \vec{H} = \vec{J}_f \longrightarrow \oint \vec{H} \cdot d\vec{l} = \underbrace{\int \vec{J}_f \cdot d\vec{s}}_{=0}$

(i) $H_\phi 2\pi s = 0 \longrightarrow \boxed{\vec{H}_\phi = 0}$



(ii) $-H_z \Delta z + H_z(z=0)\Delta z = 0$



$\longrightarrow H_z = H_z(z=0) \equiv$ constant

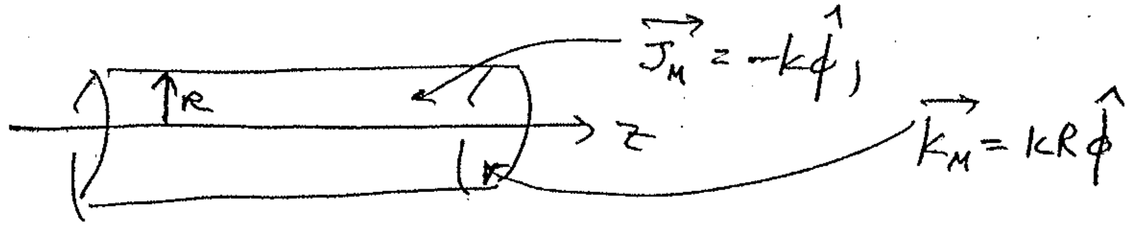
if $H_z \to 0$ at $s = \infty \longrightarrow \boxed{\vec{H}_z = 0}$

Now, $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$

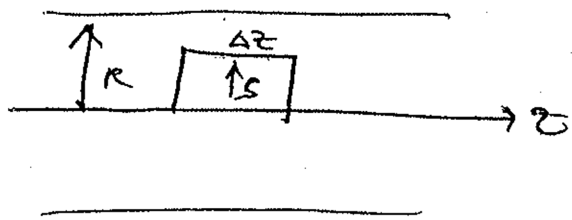
$$\longrightarrow \vec{B} = \begin{cases} 0 & , s > R \\ \mu_0 k s \hat{z} & , s < R \end{cases}$$

a) $\vec{\nabla} \times \vec{B} = \mu_0 \left[\vec{J_f} + \vec{J_M} \right] = \mu_0 \vec{J_M}$

$$\longrightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J_M} \cdot d\vec{S}$$



(i)



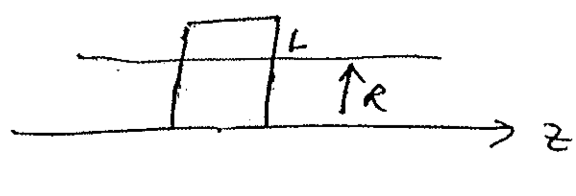
$$\oint \vec{B} \cdot d\vec{l} = -B_z \Delta z + B_z(z=0)\Delta z$$

$$= \mu_0 \oint \vec{J_M} \cdot d\vec{S}$$

$$= \mu_0 \oint (-k\, dz\, ds)$$

$$\longrightarrow (-B_z + B_z(z=0))\Delta z = -\mu_0 k \Delta z\, s \longrightarrow \boxed{\vec{B_z} = \mu_0 k s \hat{z}}$$

ii)

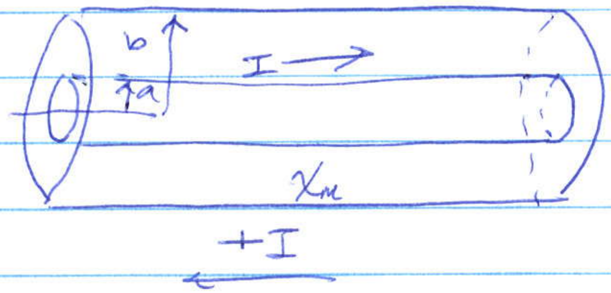


$$\oint \vec{B} \cdot d\vec{l} = -B_z(s>R)\Delta z + B_z(s=0)\Delta z = \int \vec{J_M} \cdot d\vec{S}$$

$$= 0!$$

$$\longrightarrow \boxed{B_z(s>R) = B_z(s=0) = 0}$$

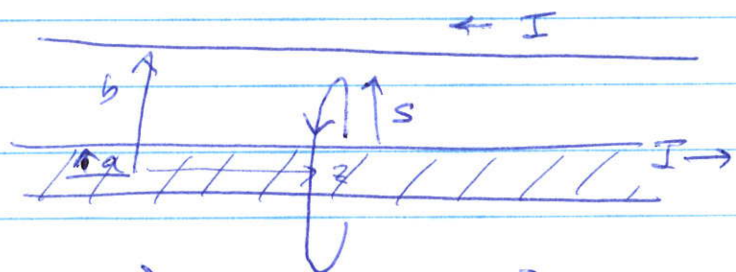
Prob 6.16



Two cylinders separated by material w/ susceptibility χ_M. A current flows down the inner cylinder, magnitude I, and returns on the outer surface.

a) Find the field in the region between a & b.

find $\vec{H}$, because it depends only on the free current, I.



a) $\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} \Rightarrow H_\phi 2\pi s = I, \quad \underbrace{\vec{H}_\phi = \frac{I}{2\pi s}\hat{\phi}}_{s=[a,b]}$

b) $\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} \Rightarrow H_\phi 2\pi s = 0, \quad s > b$

$\Rightarrow \vec{H}_\phi = 0, \quad s > b$

b) Find $\vec{M}$ & $\vec{J}_M$

$$\vec{M} = \chi_m \vec{H} = \begin{cases} \chi_m \dfrac{I}{2\pi s}\hat{\phi} & s = [a,b] \\ 0 & s > b \end{cases}$$

$$\vec{J}_M = \vec{\nabla} \times \vec{M} = \frac{\chi_m I}{2\pi}\left[0\hat{s} + 0\hat{\phi} + (0)\hat{z}\right] = 0$$

$$\vec{K}_M = \vec{M} \times \hat{n}$$

$$= \begin{cases} \chi_m \left(\frac{I}{2\pi a}\right)(\hat{z}) & , s=a \\ \chi_m \left(\frac{I}{2\pi b}\right)(-\hat{z}) & , s=b \end{cases}$$

c) Find the field using $\vec{J}_M$, $\vec{K}_M$, and I_s

The total current at $r=a$ is

$$I_a = I + \chi_m \frac{I}{2\pi a} \times 2\pi a = (1+\chi_m) I$$

The total current at $r=b$ is

$$I_b = -I - \chi_m \left(\frac{I}{2\pi b}\right) \times 2\pi b = -(1+\chi_m) I$$

(i) In the region $[a, b]$

$$B_\phi 2\pi s = \mu_0 \left[+(1+\chi_m) I\right]$$

$$\rightarrow \vec{B}_\phi = \frac{\mu I}{2\pi s} \hat{\phi} = \vec{H}_0 = \frac{I}{2\pi s}\hat{\phi}$$

✓

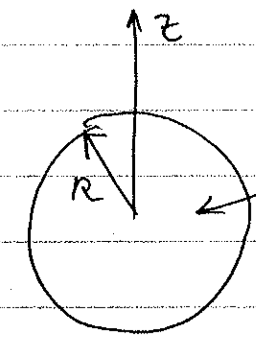
(ii) $s > b$

$$B_\phi 2\pi s = \mu_0 [0] = 0$$

$$\rightarrow \vec{B}_\phi = 0 \quad , s > b$$

33. Prob 6.18

$\vec{B}_0$ at infinity $= B_0 \hat{z}$



linear material w/ μ in sphere

Find $\vec{B}$ in the sphere.

a) $\vec{\nabla} \times \vec{H} = \vec{J}_f = 0 \longrightarrow \vec{H} = -\vec{\nabla} V_M$

and $\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot [\mu \vec{H}] = \vec{\nabla} \cdot \vec{H} = 0 \longrightarrow \nabla^2 V_m = 0$

b) Boundary Conditions

(i) at ∞, $\vec{B} = \vec{B}_0 \longrightarrow \vec{H}_0 = \dfrac{\vec{B}_0}{\mu_0}$ at infinity

$$\longrightarrow V_m(\infty) = -\frac{B_0}{\mu_0} z$$

$$= -\frac{B_0}{\mu_0} r\cos\theta$$

The general sol$^{\underline{n}}$ to the Laplace equation is

$$V_m = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

To make V_m finite at the origin,

$$\boxed{V_m^{<R} = \sum_{l=0}^{\infty} A_l^{<} r^l P_l(\cos\theta)}$$

To make $V_m \rightarrow -\frac{B_0}{\mu_0} r\cos\theta$ at ∞

$$\boxed{V_m^{>R} = \sum_{l=0}^{\infty} \frac{B_l^{>}}{r^{l+1}} P_l(\cos\theta) - \frac{B_0}{\mu_0} r \overbrace{P_1(\cos\theta)}^{\equiv \cos\theta}}$$

Boundary Conditions at surface of sphere

(a) $V_m^{>R}(R) = V_m^{<R}(R)$

$$\sum_{l=0}^{\infty} A_l^{<} R^l P_l(\cos\theta) = -\frac{B_0}{\mu_0} R P_1(\cos\theta) + \sum_{l=0}^{\infty} \frac{B_l^{>}}{R^{l+1}} P_l(\cos\theta)$$

$\underline{l=0} \rightarrow A_0^{<} = B_0^{>}/R$

$\underline{l=1} \rightarrow A_1^{<} R = -\frac{B_0}{\mu_0} R + \frac{B_1^{>}}{R^2}$

$\vdots$

$\underline{l=l} \rightarrow A_l^{<} R^l = \frac{B_l^{>}}{R^{l+1}} \rightarrow A_l^{<} = R^{-2l-1} B_>^l$

(b) $\vec{\nabla}\cdot\vec{B} = 0 \longrightarrow \Delta B_r = 0 \rightarrow \mu H_r^{<} = \mu_0 H_r^{>}$

(at $r=R$)

$$\rightarrow -\mu \frac{\partial V_m^{<}}{\partial r}\bigg|_{r=R} = -\mu_0 \frac{\partial V_m^{>}}{\partial r}\bigg|_{r=R}$$

$$\Rightarrow -\mu \frac{\partial V_m^{<}}{\partial r}\bigg|_{r=R} = -\mu\left[\sum_{l=0}^{\infty} l A_l^{<} R^{l-1} P_l\right]$$

$$-\mu \frac{\partial V_m^{>}}{\partial r}\bigg|_{r=R} = -\mu_0\left[-\frac{B_0}{\mu_0}P_1 - \sum_{l=0}^{\infty}(l+1)\frac{B_l^{>}}{R^{l+2}}P_l\right]$$

$\underline{l=0}$: $$-\mu\times 0 = +\mu_0\left[\frac{B_0^{>}}{R^2}\right] \longrightarrow B_0^{>} = 0 \rightarrow A_0^{<} = 0$$

from a)

$\underline{l=1}$: $$-\mu\left[A_1^{<}\right] = \mu_0\left[\frac{B_0}{\mu_0} + 2\frac{B_1^{>}}{R^3}\right] = -\mu\left[\frac{B_1^{>}}{R^3} - \frac{B_0}{\mu_0}\right]$$

$$\rightarrow \frac{B_1^{>}}{R^3}\left(2\mu_0 + \mu\right) = +\frac{\mu B_0}{\mu_0} - B_0 = -B_0\left(\frac{-\mu+\mu_0}{\mu_0}\right)$$

$$\rightarrow \boxed{B_1^{>} = -\frac{B_0 R^3}{\mu_0}\left(\frac{-\mu+\mu_0}{\mu+2\mu_0}\right)}$$

$$\rightarrow A_1^{<} = -\frac{B_0}{\mu_0} - \frac{B_0 R^3}{\mu_0 R^3}\left(\frac{-\mu+\mu_0}{\mu+2\mu_0}\right)$$

$$\boxed{A_1^{<} = -\frac{B_0}{\mu_0}\left(\frac{2\mu + 3\mu_0}{\mu + 2\mu_0}\right)}$$

$$\underline{l=2}:\ -2\mu A_2^{<} R = +3\mu_0 \frac{B_2^{>}}{R^4} \rightarrow A_2^{<} = -\frac{3}{2}\frac{\mu_0}{\mu}\frac{B_2^{>}}{R^5}$$

$$\xrightarrow{\text{from (a)}} = B_2^{>}/R^5$$

$$\rightarrow B_2^{>} = 0 = A_2^{<}$$

⋮ and so on

$$\rightarrow \boxed{V_m(r,\theta) = \begin{cases} -\dfrac{B_0}{\mu_0}\left(\dfrac{2\mu + 3\mu_0}{\mu + 2\mu_0}\right)\overbrace{r\cos\theta}^{z}, & r < R \\[2ex] -\dfrac{B_0 R^3}{\mu_0}\left(\dfrac{\mu + \mu_0}{\mu + 2\mu_0}\right)\dfrac{\cos\theta}{r^2} - \dfrac{B_0}{\mu_0} r\cos\theta, & r > R \end{cases}}$$

$$\vec{H} = +\begin{cases} \dfrac{B_0}{\mu_0}\left(\dfrac{2\mu + 3\mu_0}{\mu + 2\mu_0}\right)\hat{z}, & r < R \\[2ex] +\dfrac{B_0}{\mu_0}\hat{z} + \dfrac{B_0 R^3}{\mu_0}\left(\dfrac{\mu + \mu_0}{\mu + 2\mu_0}\right)\vec{\nabla}\left(\dfrac{\cos\theta}{r^2}\right), & r > R \end{cases}$$

for $r<R$, $\vec{B} = \mu \vec{H}$ and we have,

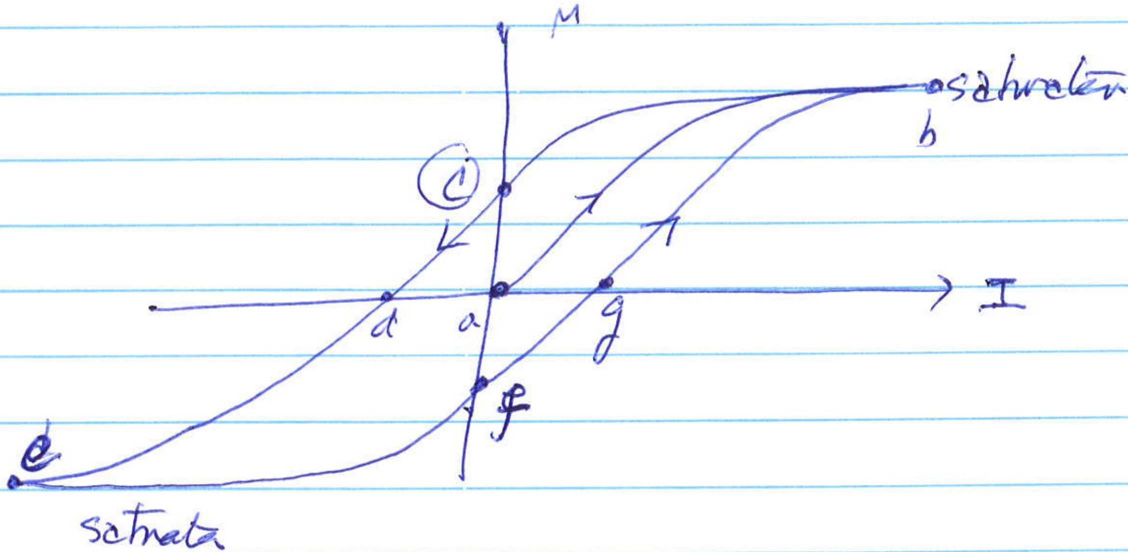
$$\boxed{\vec{B}^{<}(r,\theta) = \frac{\mu}{\mu_0} B_0 \left(\frac{2\mu + 3\mu_0}{\mu + 2\mu_0}\right)\hat{z}}$$

So, if $\mu = \mu_0$ ($\Rightarrow$ no magnetic material)

$$\longrightarrow \vec{B}^{<}(r,\theta) = B_0 \hat{z}$$

Prob 6.20

How would you go about demagnetizing a permanent magnet at point c in the hysteresis loop? That is, how could you restore it to its original state w/ $\vec{M}=0$ at $\vec{I}=0$?

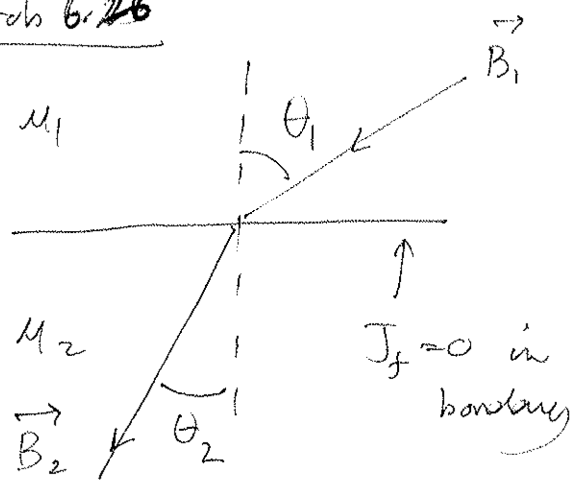


a) heat it beyond Curie temperature

b) hit it w/a "hammer" or something

c) Continue to "decrease" I to push material to its coercive field and then shut off current (increase to 0) to return to original state

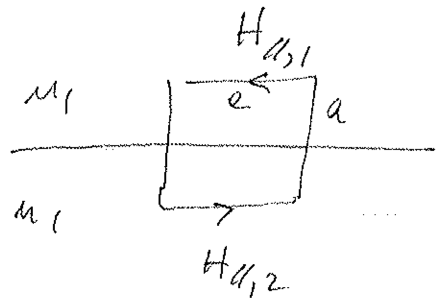
Prob 6.26



Show that $\dfrac{\tan\theta_2}{\tan\theta_1} = \dfrac{\mu_2}{\mu_1}$

(i) $\Delta B_n = 0 \implies B_{1,\perp} = B_{2,\perp}$

(ii)



$\vec{\nabla} \times \vec{H} = 0$, since $J_f = 0$

$\implies \oint \vec{H} \cdot d\vec{\ell} = 0$

and so,

$$H_{\parallel,1}\,\ell - H_{\parallel,2}\,\ell = 0$$

$$H_{\parallel,1} = H_{\parallel,2}$$

and for a linear medium

$$\mu_1^{-1} B_{\parallel,1} = \mu_2^{-1} B_{\parallel,2}$$

(iii) $\tan\theta_1 = \dfrac{B_{\parallel,1}}{B_{\perp,1}}$, $\tan\theta_2 = \dfrac{B_{\parallel,2}}{B_{\perp,2}}$

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{B_{\parallel,2}}{B_{\parallel,1}}\,\frac{B_{\perp,1}}{B_{\perp,2}} = \left(\frac{\mu_2}{\mu_1}\right)\left(1\right) = \frac{\mu_2}{\mu_1}$$